A Decimetre Height Reference Surface (HRS) for the European Vertical Reference System (EVRS) based on the DFHRS Concept

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INTRODUCTION

European GNSS- Services
RTCM-Phase Corrections
GNSS cm-Positioning

SAPOS®

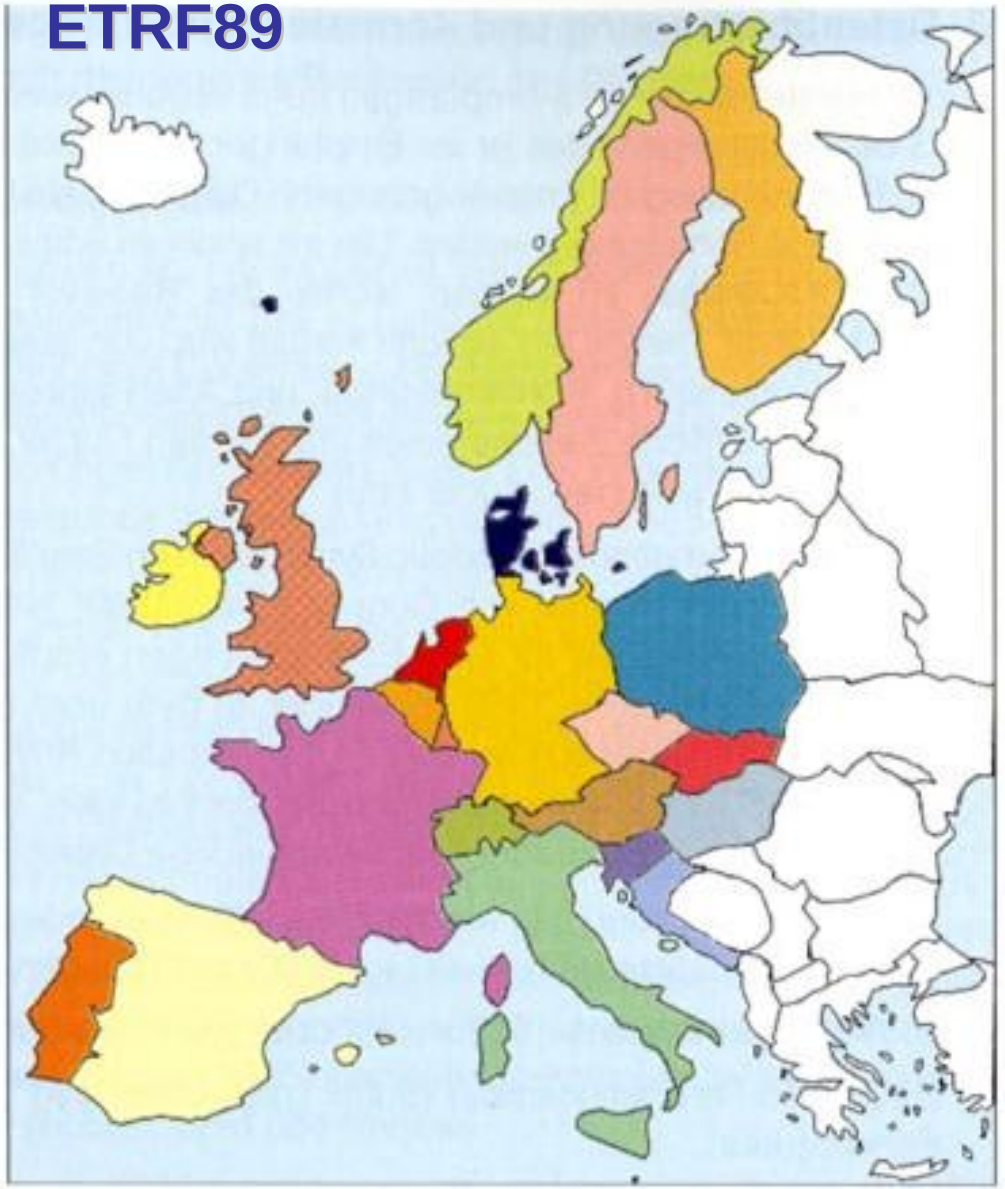
ascos™
ruhrgas positioning services

SWIPOS®

SwissSat®

SWEPOS®

EUPOS®
European Position Determination System



INTRODUCTION

1st Present Transformation Problem

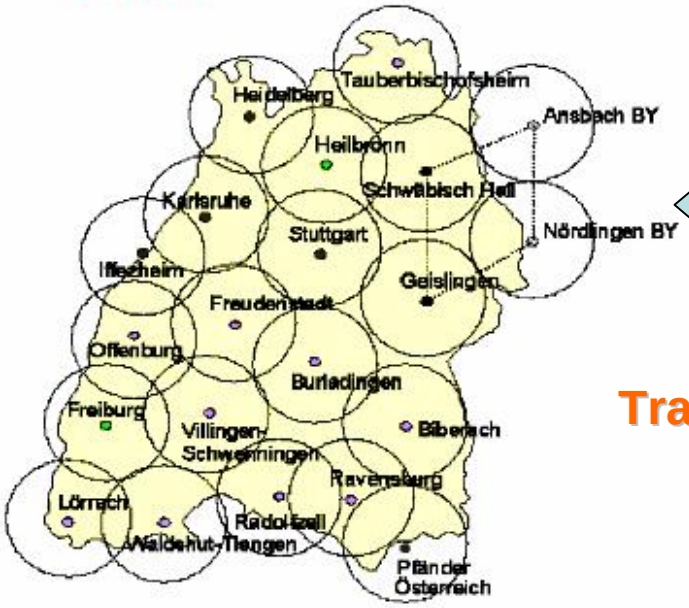
Classical National Datum Systems obsolete

Modern Geo-Referencing is related to ETRS89 / ITRF

Economical Online Positioning in homogeneous Datum

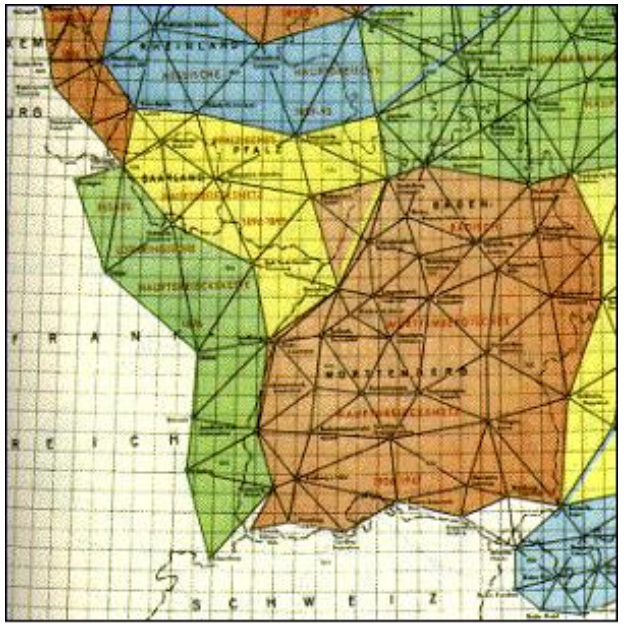
political task of State Land Services of all Nations

SAPOS®- Referenzstationen



ETRS89-Datum

General strict Transformation



Old Datum
(e.g. „DHDN“ in Germany)

INTRODUCTION

Strict 3D-Trafo in (B,L, (h))

Residuals Germany

without Patching.

Mean Residual:

1.49 m

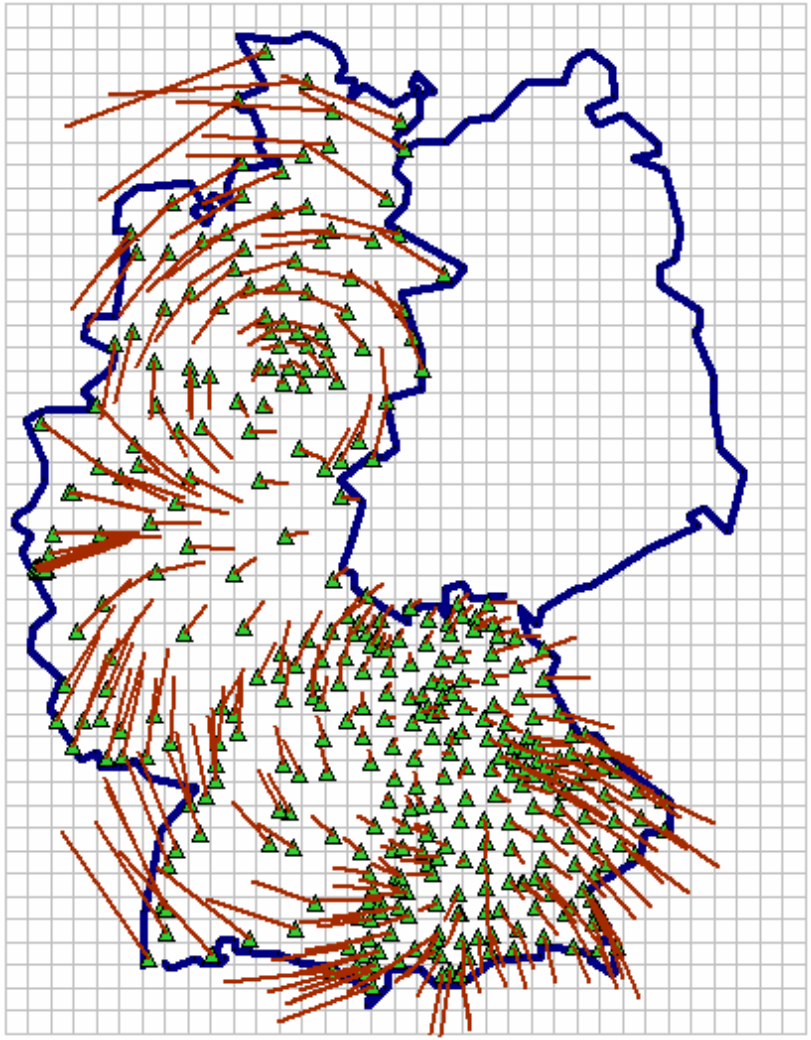
Max. Residual:

2.43 m

Longwaved

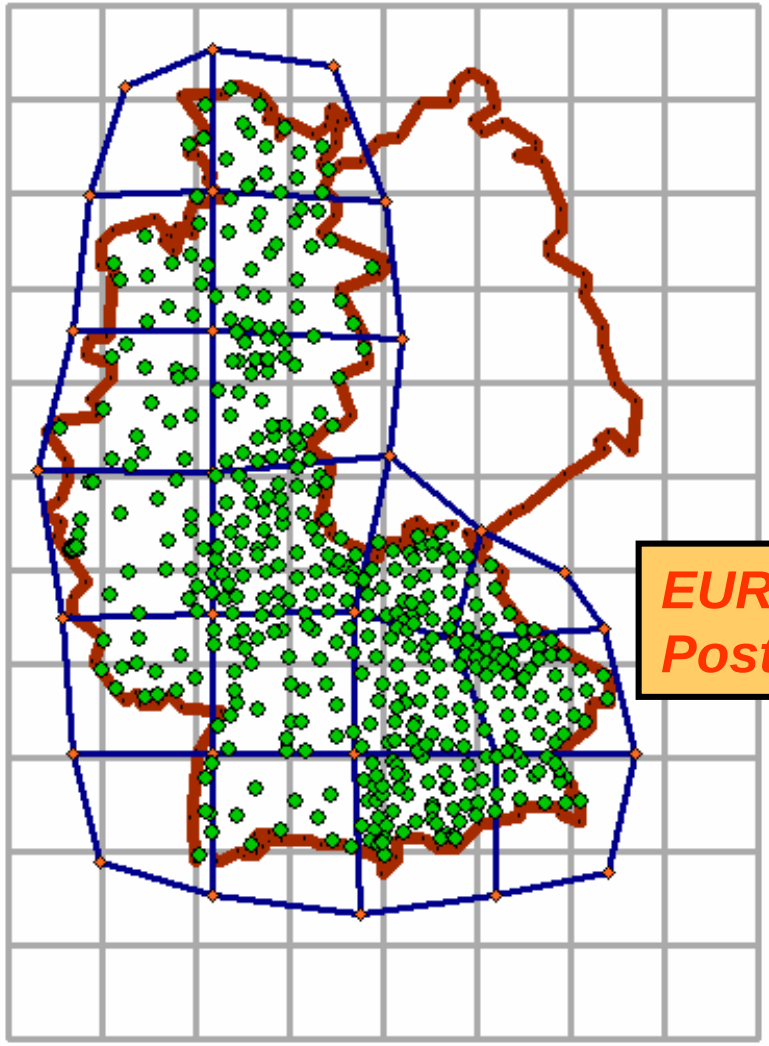
Quasi-Systematic

Errors („Weak Shapes“)

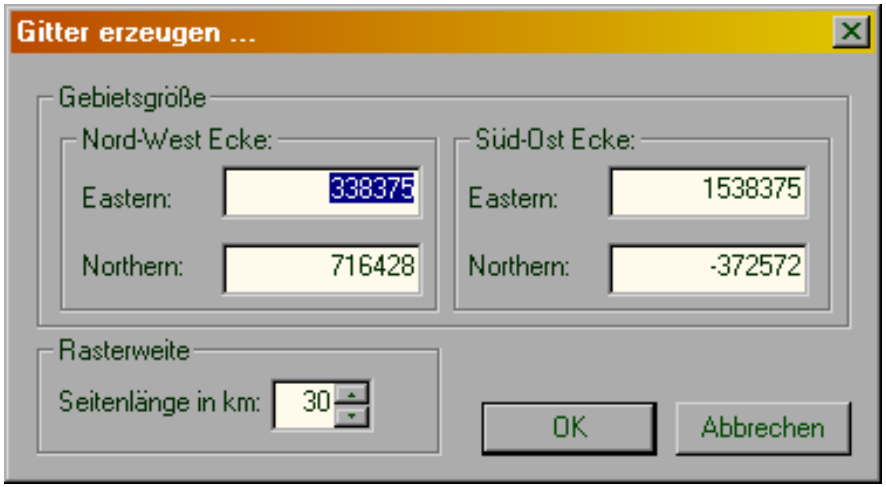


INTRODUCTION

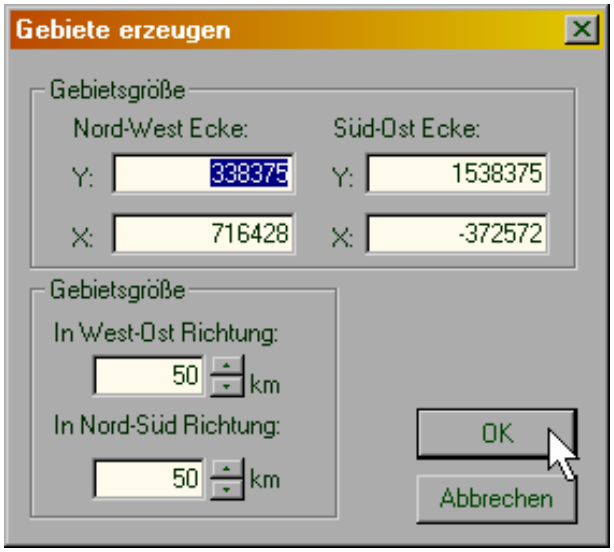
COPAG Concept



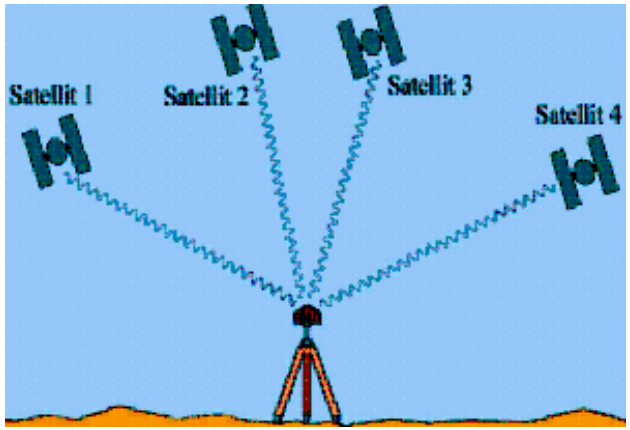
Mesh Generation



EUREF2004
Poster-Presentation

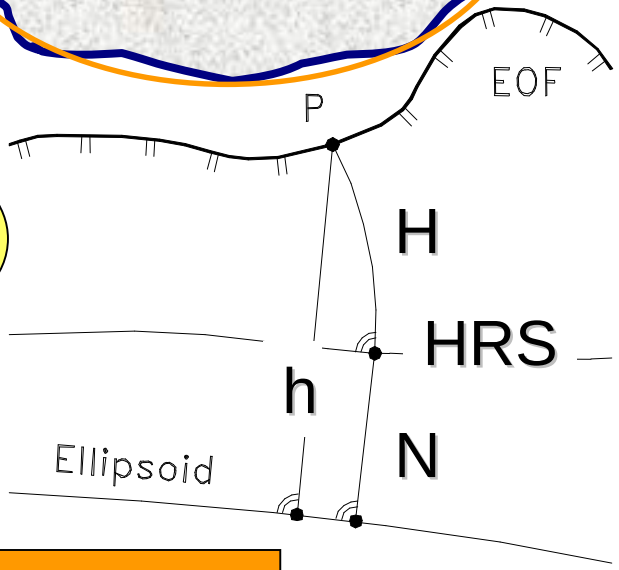


2nd Present Transformation Problem

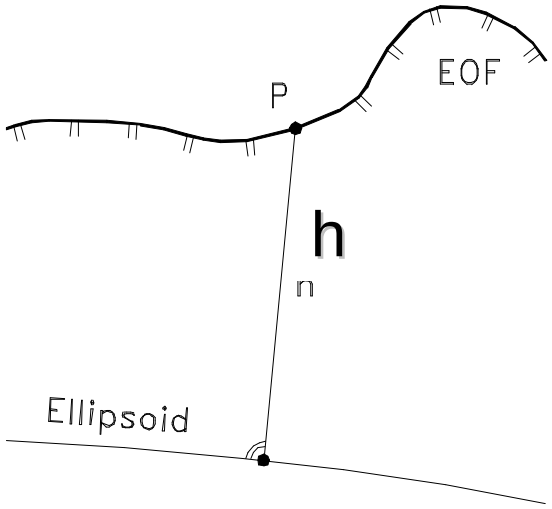


+/- 100 m

„Geoid“ = HRS
Ellipsoid



GNSS Heighting
„H from h- GNNS“



H = h - N(B,L,h)

"Geoid" or better : HRS

European Vertical Reference System (EVRS)

Discrete Points of the HRSurface (B,L,h; H)

UELN 95/98
Adjustment by Geopotential Numbers

Datum W_0 and C_0
Amsterdam
„NAP“

Mean Accuracy
+/- 3.0 cm

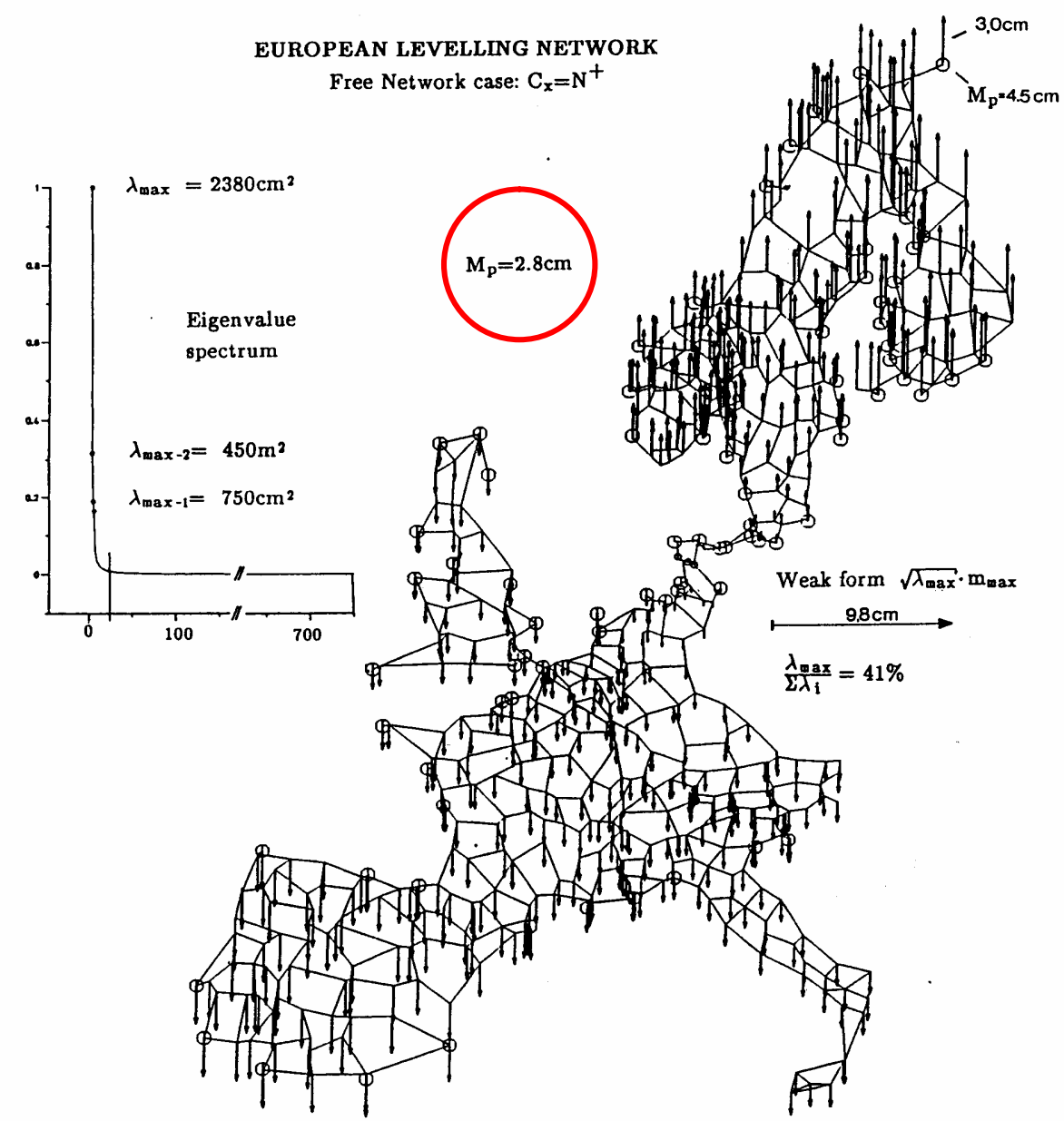
Error Budgets

European Vertical Reference System (EVRS)

Analysis of „Weak Shapes“

Jäger, 1989
IAG-Symposium
Edinburgh

Mean Accuracy:
3.0 cm



Spectral Analysis „Weak Shapes‘ Analysis “

$$r = \sum_{i=1}^{n} r_i = \sum_{i=1}^{n} c_i \cdot m_i$$

{ m_i -Spectral Carrier Functions /Basis
c_i - Stochastical Amplitudes } $C_c = \Lambda$

$$C_x = \sum_{i=1}^{n}\sum_{j=1}^{n} \Lambda_{ij} \cdot m_i \cdot m_j^T = (A^T \cdot C_l^{-1} A)^{-1}$$

Influence of random Observation Errors
C_l

Idea:

$$E\left(\frac{r_i^T \cdot r_i}{r^T r - r_i^T r_i}\right) = \text{Max}$$

(Jäger, 1988)

Solution:

m_i - Eigenvectors of C_x
$E(c_i c_i) = l_i$ Λ - Diagonal Matrix of Eigenvalues of C_x

$$\sqrt{\lambda_i} \cdot m_i = \text{„Weak Shapes“}$$

Additionally: Latent „Weak Shapes“ due to neglected Physical Correlations

$$\Delta C(i,j) = k_i \cdot [\sigma_a^2 \cdot c_a(t_i - t_j)] \cdot k_j = k_i \cdot C_{\delta_2\delta_2} \cdot k_j$$

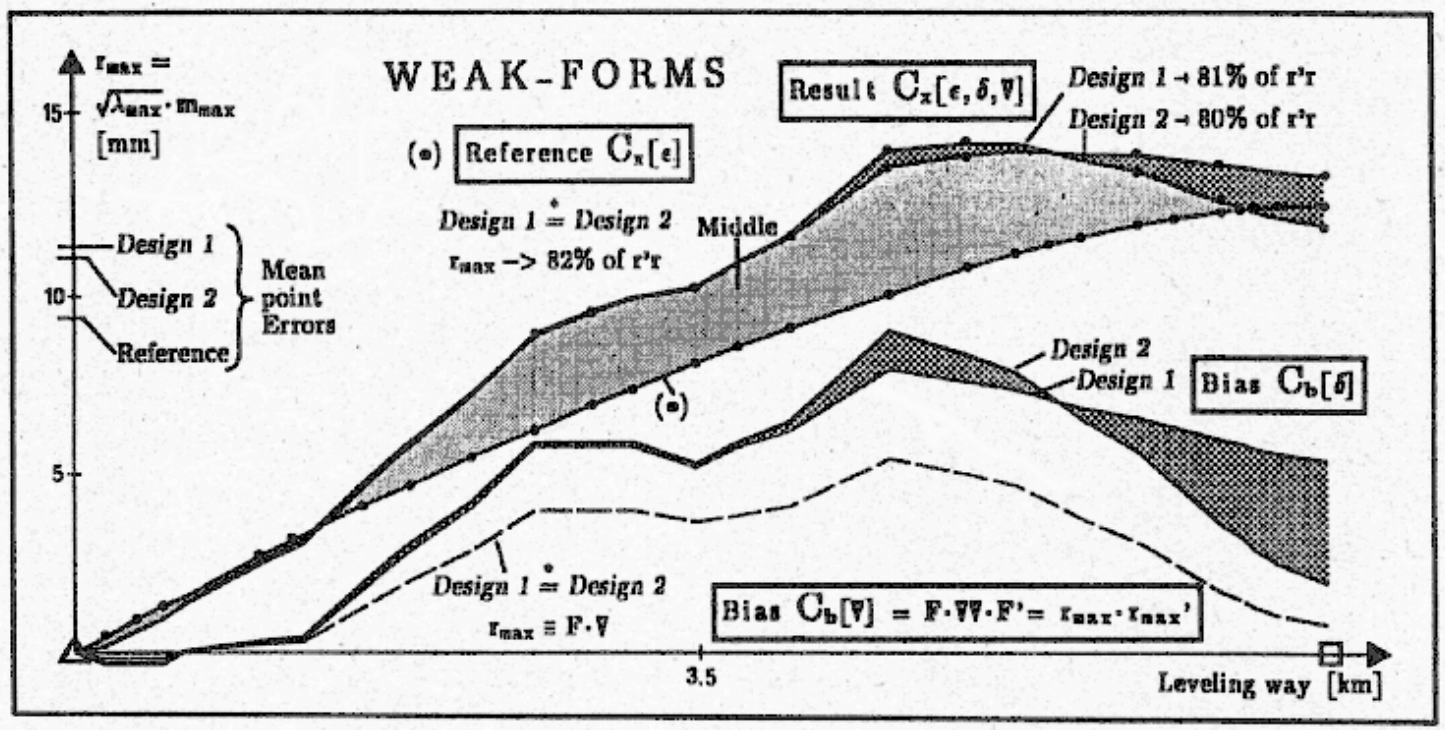


Fig. 4
Comparative study by leveling–line (fig. 3): Differences in shape and amount of principal bias' and weak–forms r_{max} for DESIGN 1 and DESIGN 2. Comparison to the time–invariant reference of pure random errors ϵ.

Jäger and Leinen (1992)

6th Int. Satellite Symposium

Ohio, USA

$$[(C_{\hat{u}\hat{u}} - C_{\hat{u}\hat{u}}) - m_i \cdot C_{\hat{u}\hat{u}}] \cdot m_i = 0$$

Error Budgets

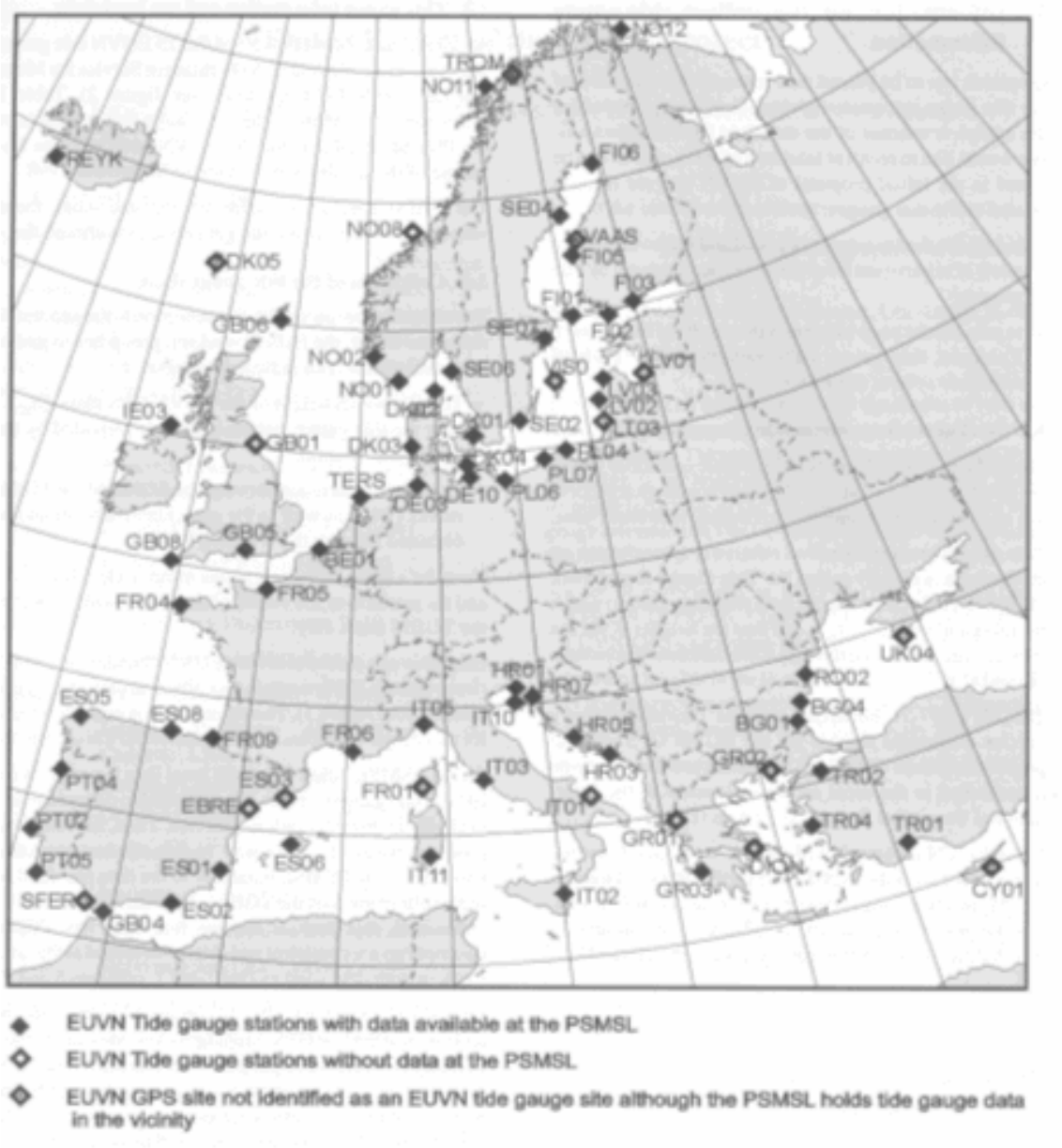
European Vertical Reference System (EVRS)

Planned:

Increasing the final EVRS Accuracy on introducing

Tide Gauge Points as further Fix Points

Mean Accuracy: 1.5 cm



◆ EUVN Tide gauge stations with data available at the PSMSL

◇ EUVN Tide gauge stations without data at the PSMSL

◈ EUVN GPS site not identified as an EUVN tide gauge site although the PSMSL holds tide gauge data in the vicinity

Vertical Reference Surface for Europe

Up to now

EGG97

European Gravimetric Geoid97

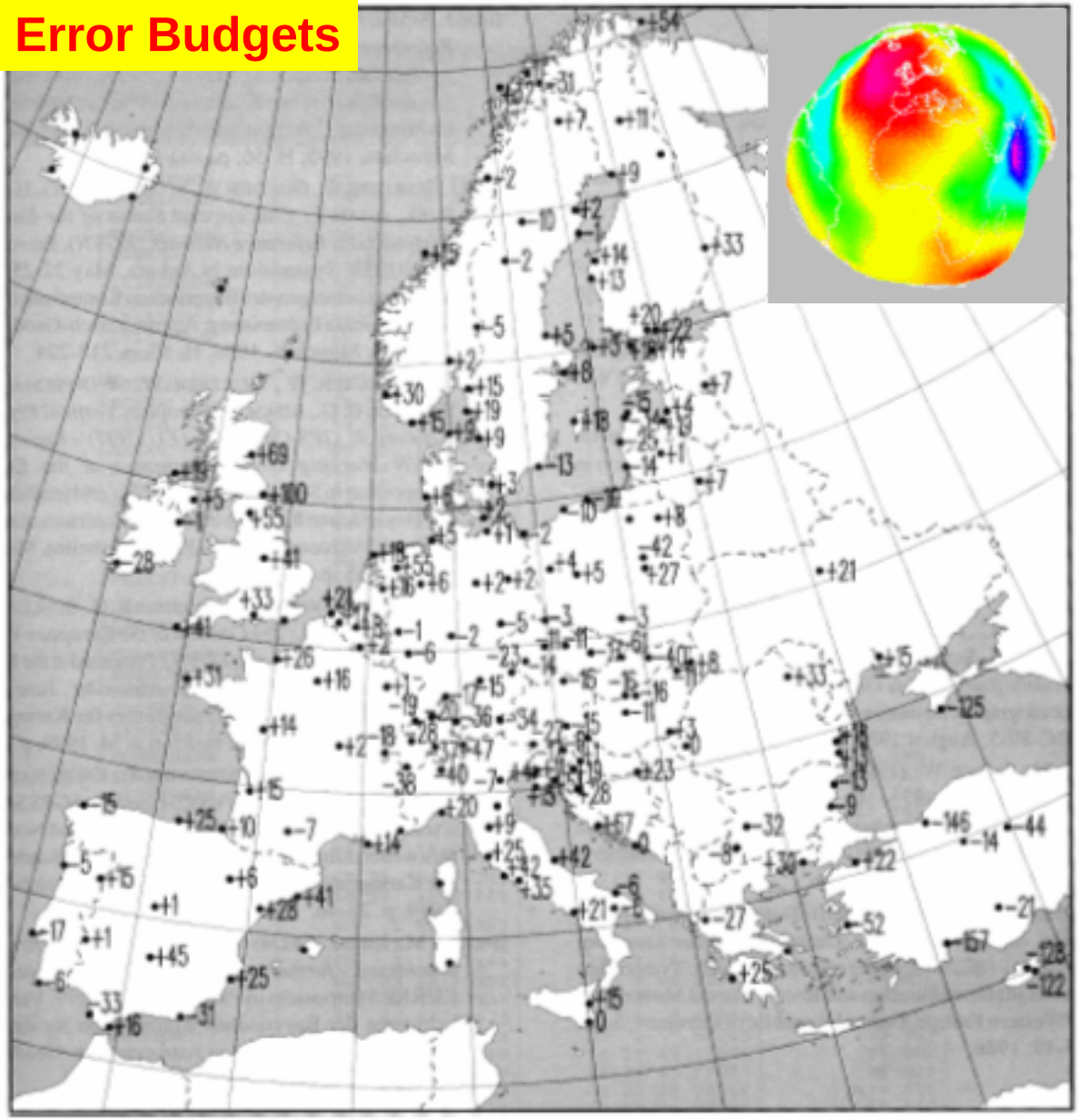
1_cm geoid (short-wave)

Quasi-Systematic Errors
0.1 – 1.5 m !

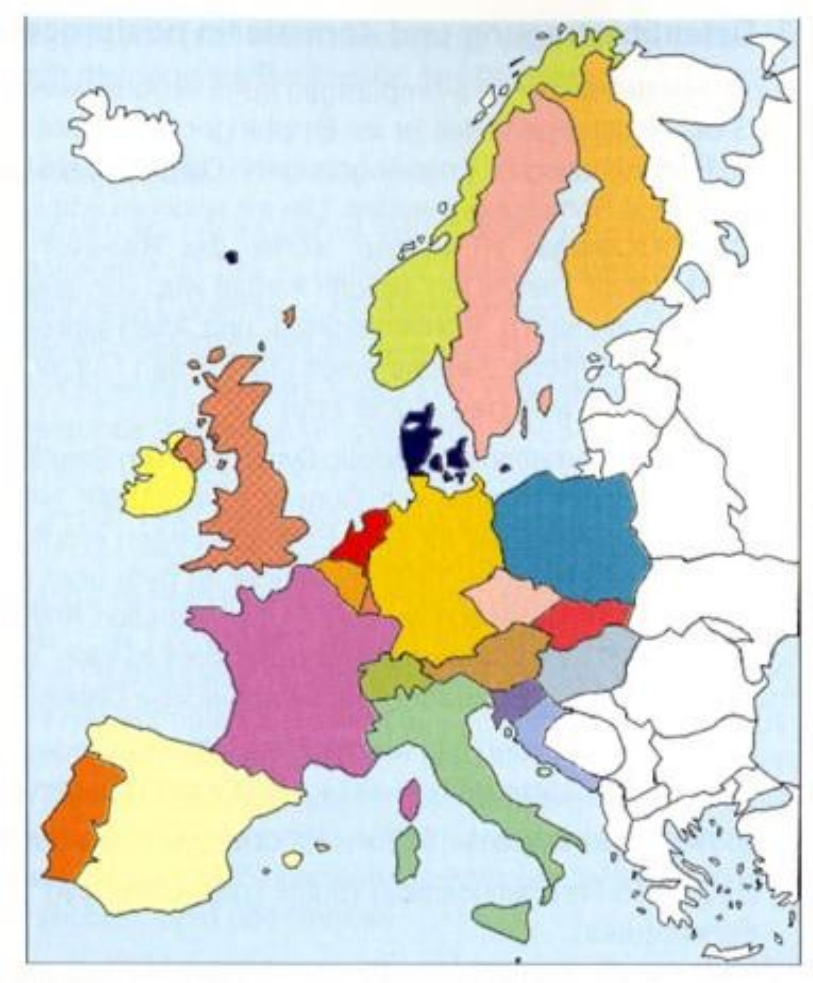
„Weak Shapes“
Long-Waved Domain

EGG97 = Non Fitted QGeoid

Error Budgets



Height Reference Surface for the EuropeanVRS



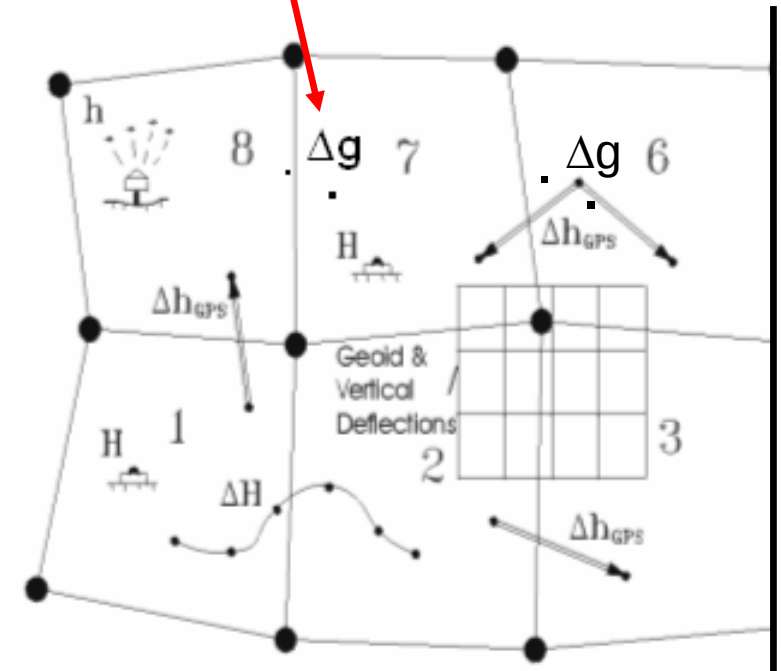
GNSS-Services in Europe

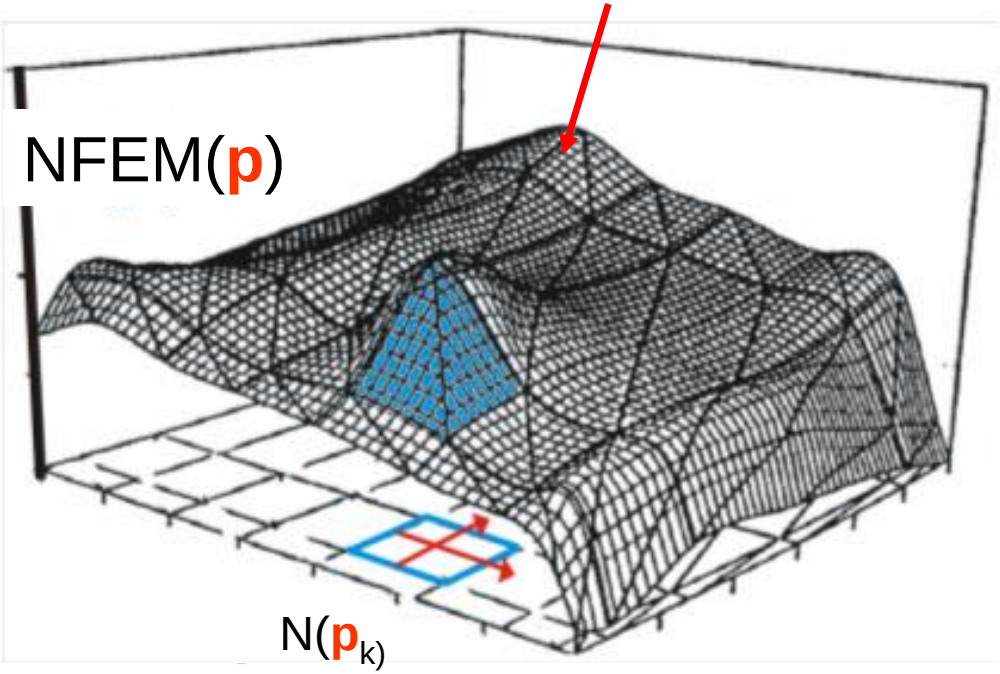
EUREF Resolution No. 4, Dubrovnik 2001

The IAG Subcommission for Europe (EUREF) recognising the European Vertical GPS Reference Network (EUVN) with its GPS-derived ellipsoidal heights and levelled connections to UELN, – the definition of the European Vertical Reference System EVRS with its first realisation UELN 95/98, called EVRF2000, considering – this implicit pointwise realisation of a European geoid consistent with both ETRS89 and EVRS, – the existence of a large number of regional and local geoids in Europe, – the urgent need by the navigation community for a height reference surface, asks its Technical Working Group and the European Sub-commission of the IAG IGGC (International Gravity and Geoid Commission) to take all necessary steps to generate a ***European "geoid model" of decimetre accuracy consistent with ETRS89 and EVRS.***

Digital-FEM-HRS (DFHRS) Approach

Idea: "All observations / data types related to the parameters p of a unique continuous one-layer FEM representation NFEM(p) of the HRS"





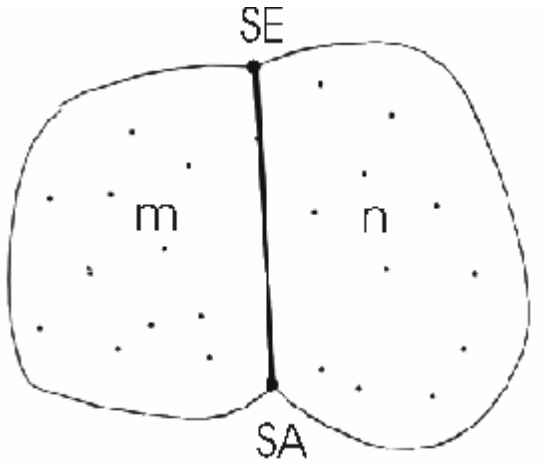
$$NFEM(\mathbf{p}) =: \left\{ N(\mathbf{p}_k) = \sum_{i=0}^{n} \sum_{j=0}^{n-i} p_{ij,k} \cdot y^i \cdot x^j \quad ; Mesh\ \ Polynomial\ , Mesh\ \ k \right\}$$
$$+ \text{Continuity Conditions along the Mesh borders}$$

$N(p_k)$= Local Taylor-Series of HRS

Continuous FEM Representation of HRS

$$\begin{bmatrix} \Delta x_{m,n} \\ \Delta y_{m,n} \\ \Delta N_{m,n}(y,x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ N(\mathbf{p}_n, y, x) - N(\mathbf{p}_m, y, x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=0}^{l} \sum_{j=0}^{l-i} (a_{ij,n} - a_{ij,m}) \cdot y^i \cdot x^j \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ \Delta N_{n,m} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=0}^{l} \sum_{j=0}^{l-i} (a_{ij,n} - a_{ij,m}) \cdot y^i \cdot x^j \end{bmatrix}$$



$$\sum_{i=0}^{l} \sum_{j=0}^{l-i} (a_{ij,n} - a_{ij,m}) \cdot (y_{sa} + t \cdot (y_{se} - y_{sa}))^i \cdot (x_{sa} + t \cdot (x_{se} - x_{sa}))^j = \Delta N_{m,n}(t) \equiv 0!$$

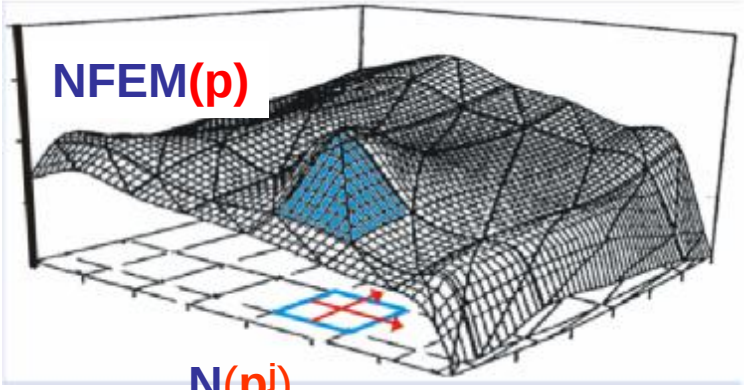
$$\Delta N_{m,n}(t) = c_0 + c_1 \cdot t + c_2 \cdot t^2 + \mathbf{L} + c_k \cdot t^k = \sum_{k=0}^{l} c_k \cdot t^k \equiv 0!$$

$$\Rightarrow \quad c_k(\mathbf{p_m}; \mathbf{p_n}; y_{sa}, x_{sa}; y_{se}, x_{se}) = 0, \quad k = 0, l$$

Digital FEM Height Reference Surface (DFHRS)- Concept

DFHRS – Approach

Complete New Computation of continuous HRS (repres. by p and D m)!



$$h_{GNSS} + v = H + NFEM(p) - h_{GPS} \cdot D\,m$$

$$N_G{}^{j} + v^{j} = NFEM(p) + \partial N_G(d^{j})$$

$$Dg + v = Dg(p)$$

$$H + v = H$$

$$\xi^{j} + v = -F_B / M(B) \cdot p + \partial \xi (d_{x,h})^{j}$$

$$\xi h^{j} + v = -F_L/(N(B)\cdot\cos(B)) \cdot p + \partial \eta (d_{x,h})^{j}$$

$$\frac{a}{4\cdot\pi\cdot\gamma(B)} \iint_{\sigma} Dg \cdot S(\psi) d\sigma + v = NFEM(p)$$

<= Any number Geoidmodels/Patches
(Existing Regional Geoids; EGG97)

<= Gravity

<= GPS/Levelling Fitting Points

<= Sets of Deflections from Vertical

(Modern Astro-Geod. Zenith Cameras)

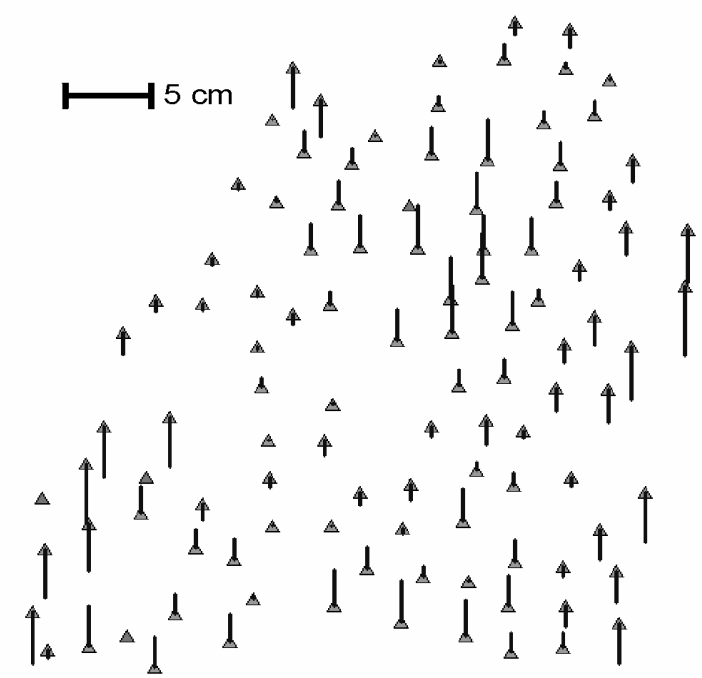
DFHRS Approach - „Patching“

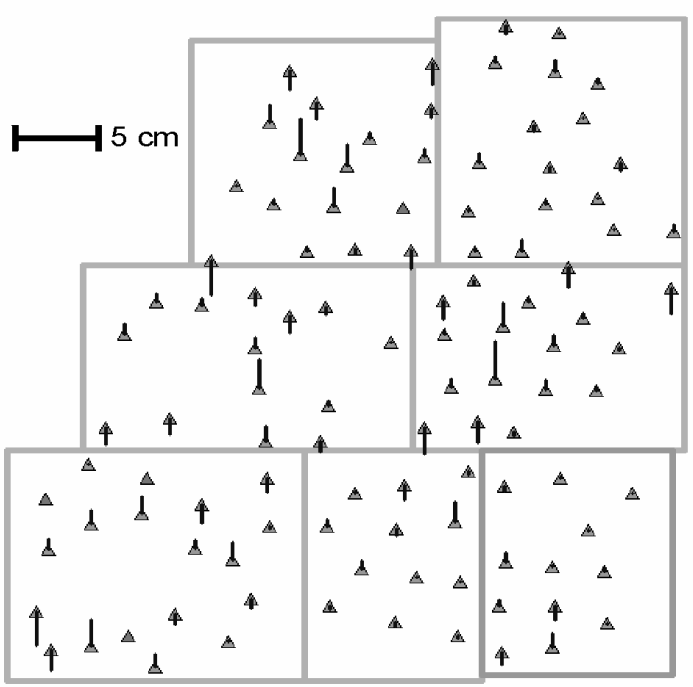
$$NG'_j + v_j = NFEM(p) + \partial NG(d_j)$$

Residuals of a Geoid Model **NG‘**

One Datum

7 „Patches“





Height Reference Surfaces and „Geoidfitting“

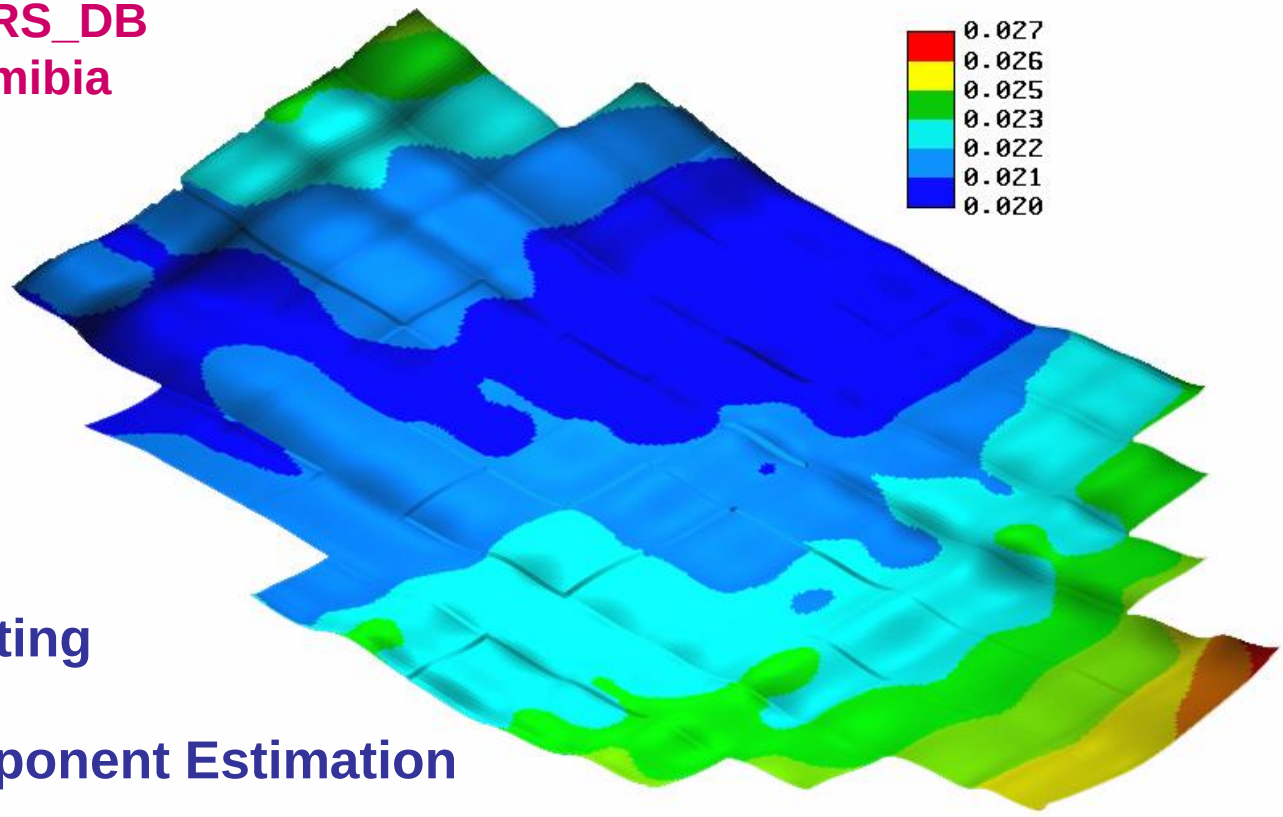
Name	Principle, Formulas	Basic Repre-sentation	Geoid-Fitting Standard	Fitted HRS Represent ation
Spherical Harmonics Series	$N = \left\{ \frac{GM}{\gamma(\varphi)\cdot r}\sum_{n=2}^{N}\left(\frac{a_{GRS80}}{r}\right)^{n} \sum_{m=0}^{n}(\delta C_{nm}\cdot\cos m\lambda + \delta S_{mn}\cdot\sin m\lambda)\cdot P_{nm}(\sin\varphi) \right\}$	Continuous Surface	$N_{fitted} = N + N_{trend} + N_{colloc}$	Grid
Gravimetric Geoid / HRS	$N = \frac{a_{GRS80}}{\gamma(\varphi)\cdot 4\pi}\iint_{\sigma} S(\psi)\cdot(\Delta g - \Delta g_{Ref})\cdot d\sigma$	Grid	$N_{fitted} = N + N_{trend} + N_{colloc}$	Grid
Point Mass Modelling	$N = \frac{1}{\gamma(\varphi)}\cdot(T(\delta C_{mn},\delta S_{nm}; M_i(x,y,z))$	Continuous Surface	$N_{fitted} = h - N$ Direct fit	Grid
DFHRS Local mesh-wise Taylor Series. Any Area Size	$N = \frac{T}{\gamma(\varphi)} =$ $NFEM(\mathbf{p}) = \left\{ N(\mathbf{p}_j) = \sum_{i=0}^{N}\sum_{k=0}^{N-i} p_{ik,j}\cdot x^i y^k \text{ and } C(\mathbf{p}_m\mathbf{p}_n) \right\}$	**Continuous Surface**	$N_{fitted} = h - N$ **Direct fit**	**Continuous Surface** (Grid)

DFHRS_DB - QUALITY Proof

„Accuracy Surface“ based on

Covariance matrix of DFHRS parameters (p,Δm)

<_3_cm DFHRS_DB
Windhuk, Namibia
EGM96

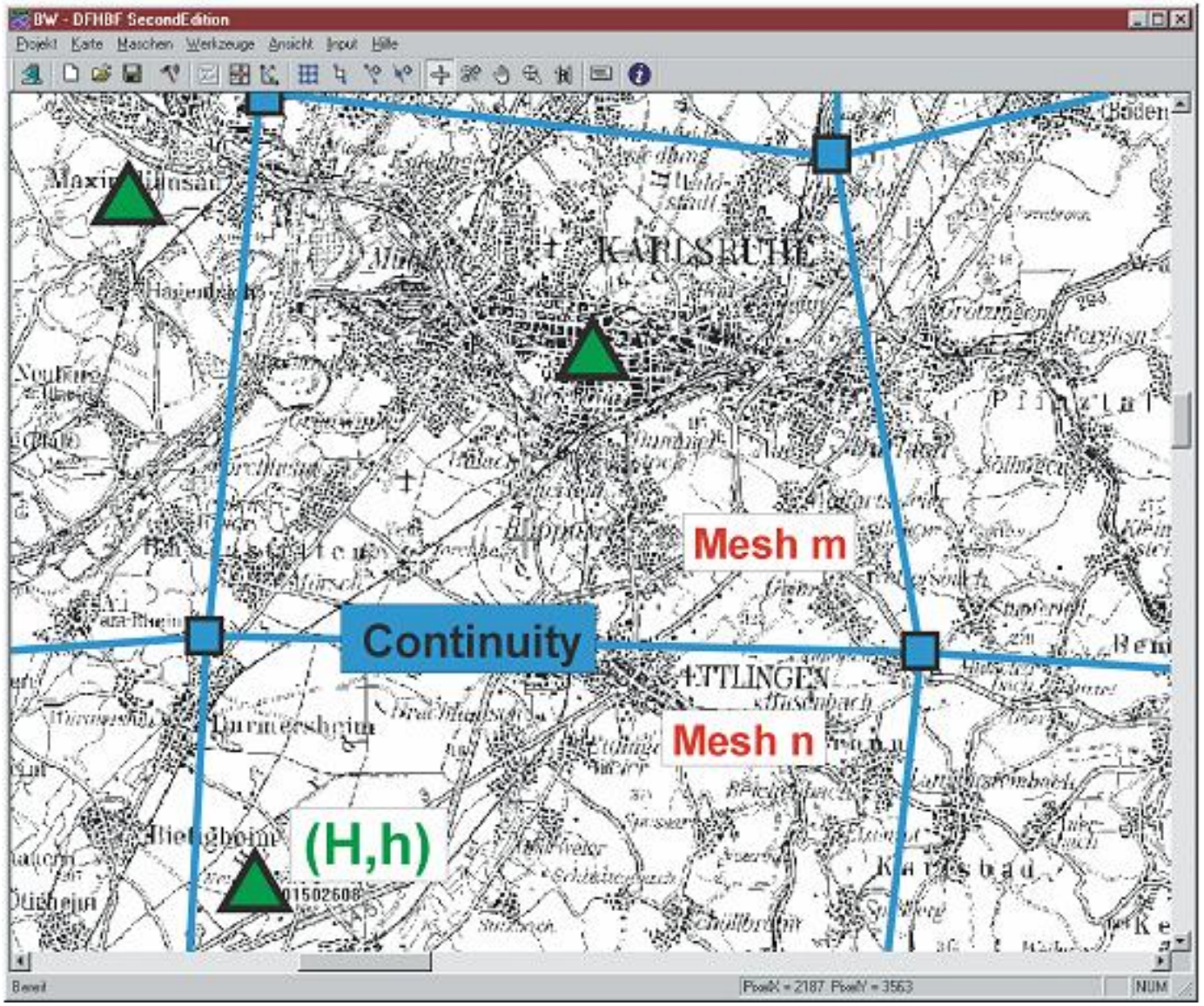


+ Statistical Testing

+ Variance Component Estimation

DFHRS Software

- Map Back Ground
- Identical Fitting Points (B,L,h;H)
- Meshes
- Residuals

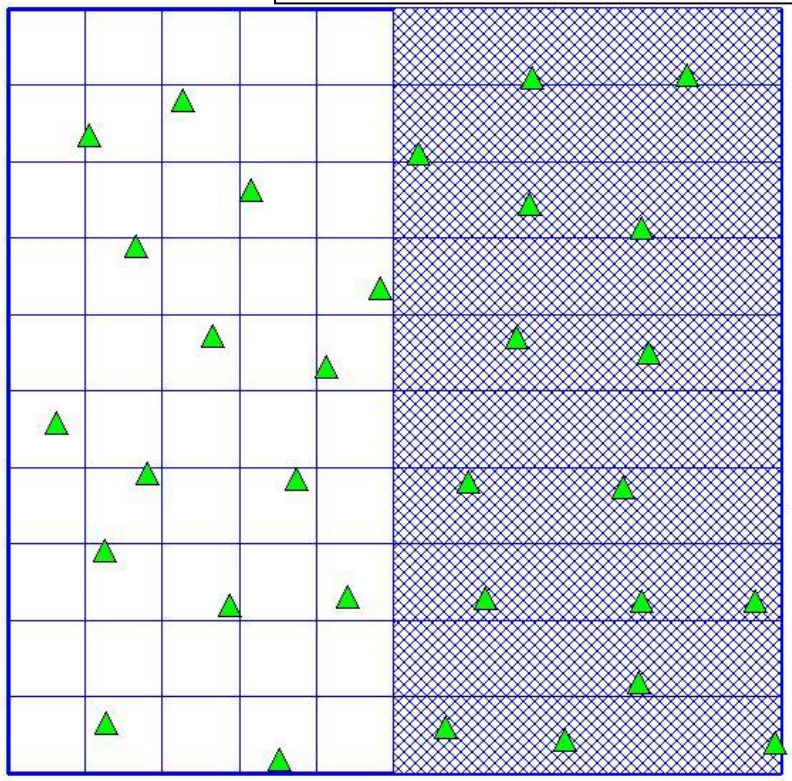


DFHRS Software

- *University of Applied Sciences (FH) Karlsruhe*
- *SLS Baden-Württemberg*
- *SLS Hessen*
- *SLS Rheinland-Pfalz*
- *SLS Riga, Latvia*
- *University of Federal Forces Munich*
- *University of Darmstadt*
- *::*

Any number of different geoid models N may be introduced Patch-wise

$\P N_G(d_j)$



DFHRS Software

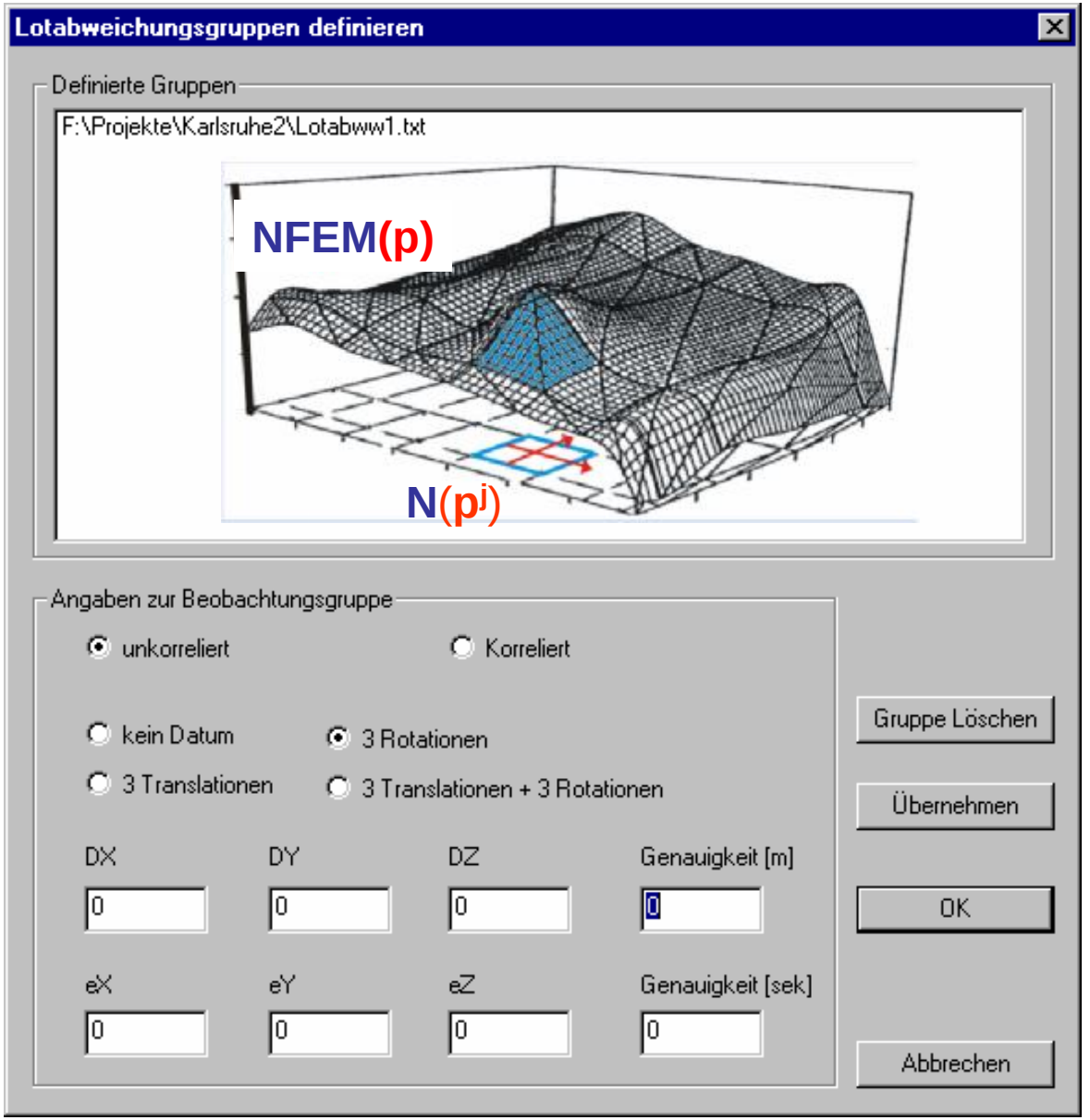
Any number of observation groups of

- **Geoidmodels**

and

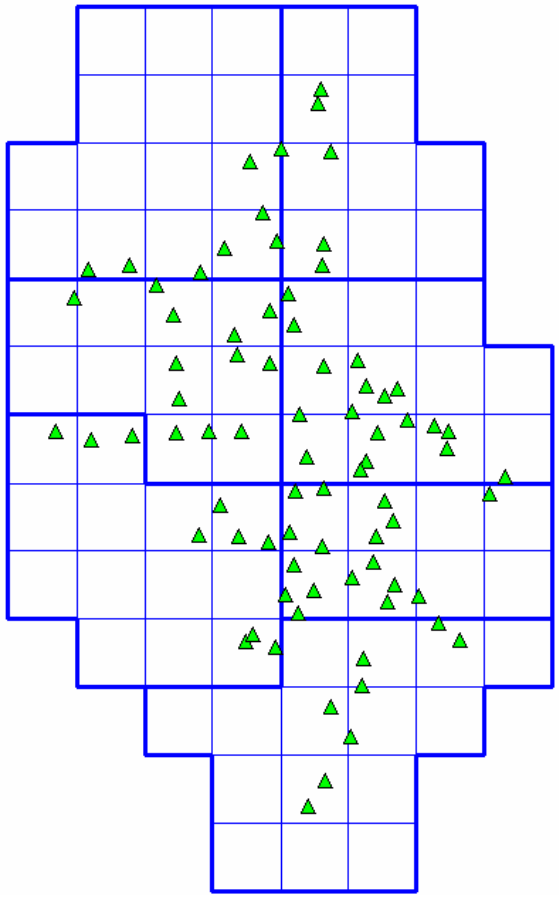
- **Deflections of the Vertical**

may be introduced simultaneously. The groups may refer to different datum systems. Datum parameters may be introduced a prioiri information.



DFHRS_DB Design Parameters

<_3_cm DFHRS_DB
Windhuk, Namibia
EGM96



Meshsize (p=3)

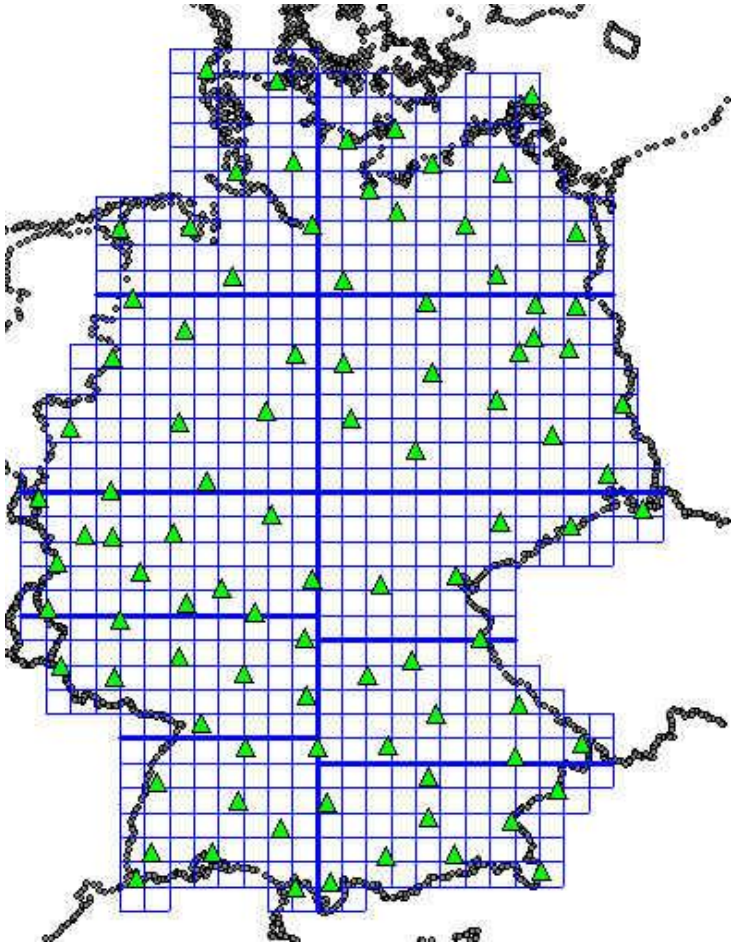
- 20-30 km : HRS approximation error < (5-10) cm
  - 10 km: HRS approximation error <1 cm
  - 5 km: HRS approximation error < 0.5 cm

Fitting Point Density (< 10 mm points, EGG97)

- 50 points per (100 km x 100 km): <_1_cm DFHRS_DB
- 10 points per (100 km x 100 km): < 3_cm DFHRS_DB
- 3-4 points per (100 km x100 km): < 5-10_cm DFHRS_DB

DFHRS_DB Design Parameters

Design Studies < 5 - 10_cm DFHRS Germany



Patch-Size (EGG97)

- 30 - 40 km for a < 1_cm DFHRS_DB
- 50 – 60 km for a < 3_cm DFHRS_DB
- 300 km for a < 10_cm DFHRS_DB

(3-5) points per patch

< 10cm DFHRS Europe - Design of Fitting Points

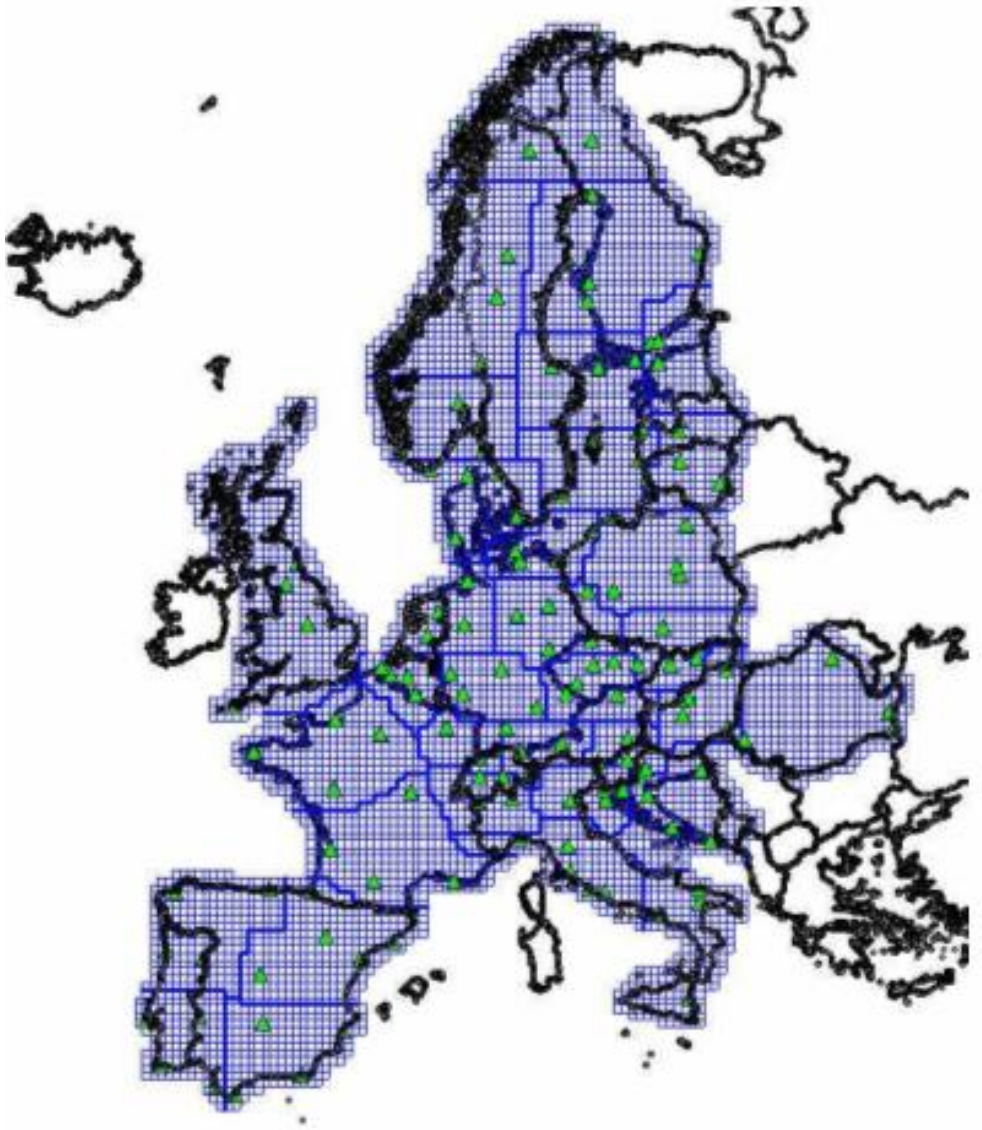
ETRS89/EVRS

„GPS/Levelling Points of EUVN“

Fitting Points NFEM(p) =: h - H

used for the the 1st version

< 10_cm DFHRS Europe

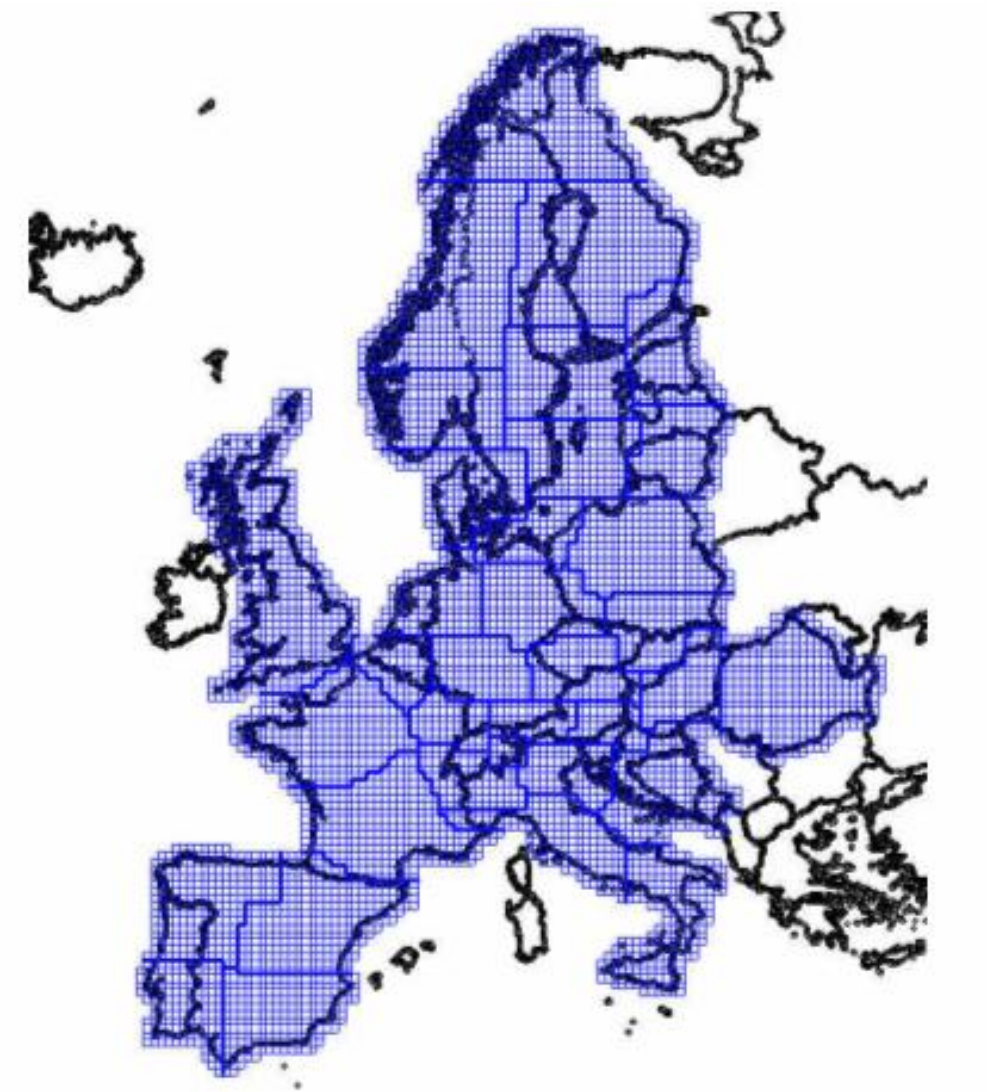


< 10 cm DFHRS_DB Europe - Patch Design

HRS Polynomial Degree N=3

30 km Meshsize

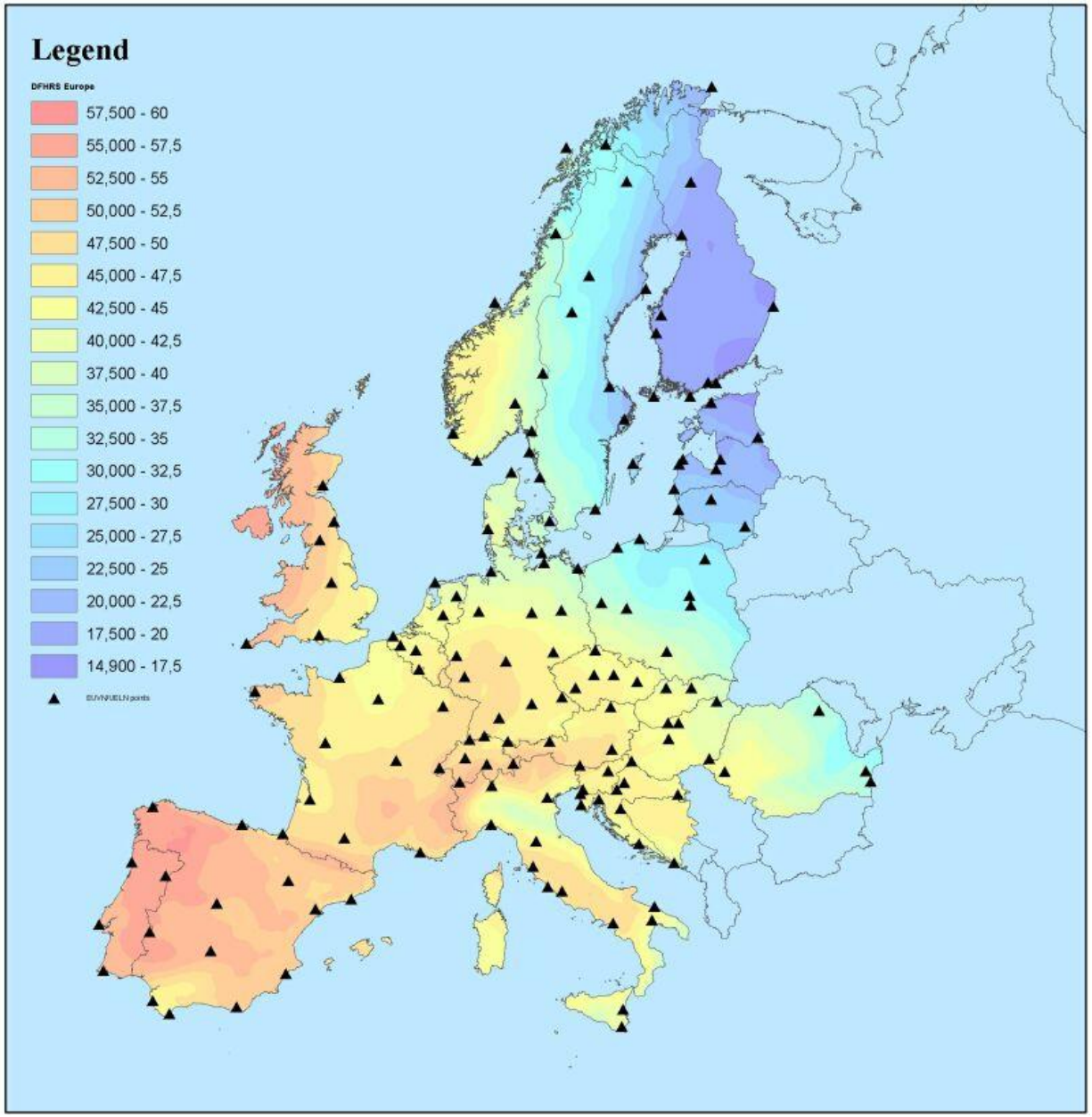
34 Geoid-Patches
Patch Size
100 – 800 km
At least (3-4) Fitting points per patch



< 10 cm
DFHRS_DB
Europe

Isolines

30 km
FEM
Meshes



<_10_cm DFHRS_DB - Indepent Quality Control

<1_dm EVRF2004 (Present Version, 35 km meshes, 34 Patches)

	Austria	Germany	Estonia	Latvia	Lithuania	Switzerland
Number of unused control points	9	95	21	25	46	13
RMS [cm]	7.5	4.2	8.8	9.2	6.8	7.0

< 10 cm DFHRS_DB USA

<_10_cm DFHRS_DB Europe – Fitting Point Design

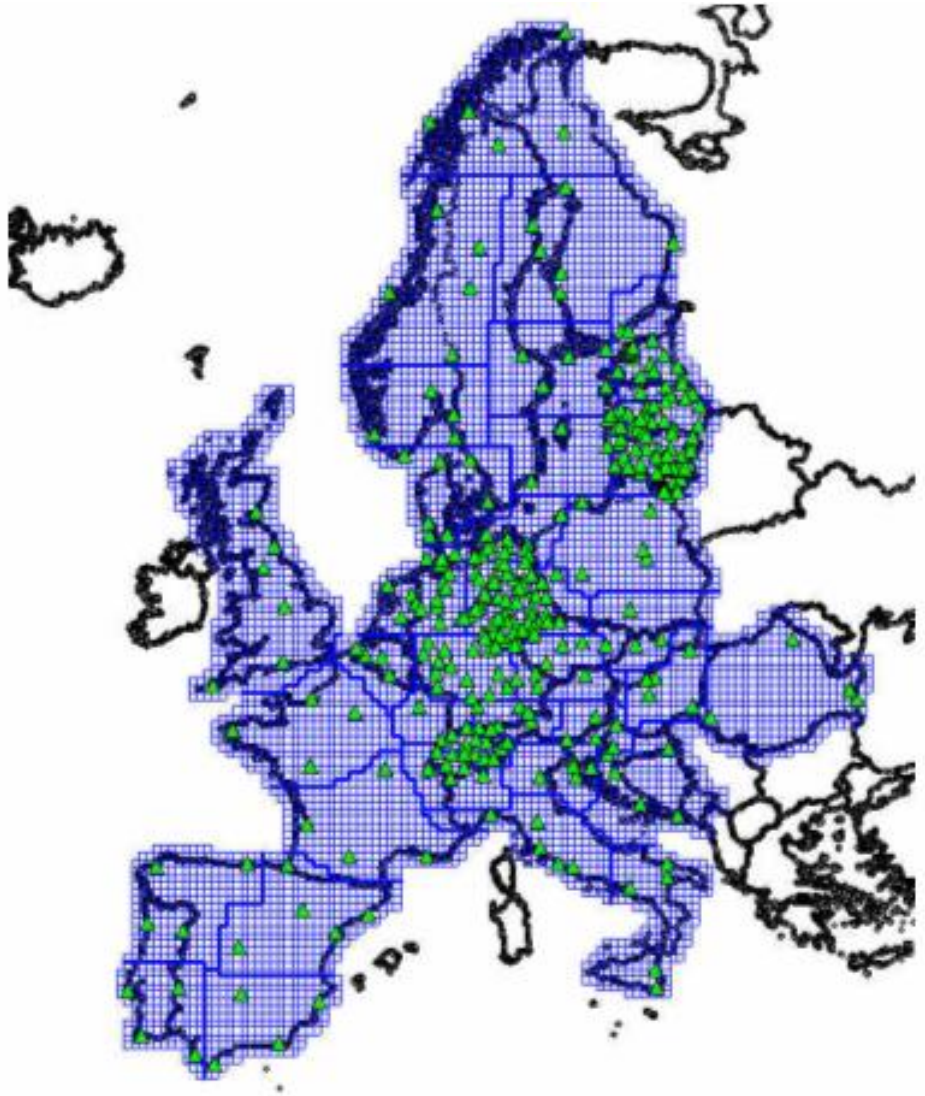
Present Data

ETRS89/EVRS
Fitting Points
NFEM(p) =: h - H

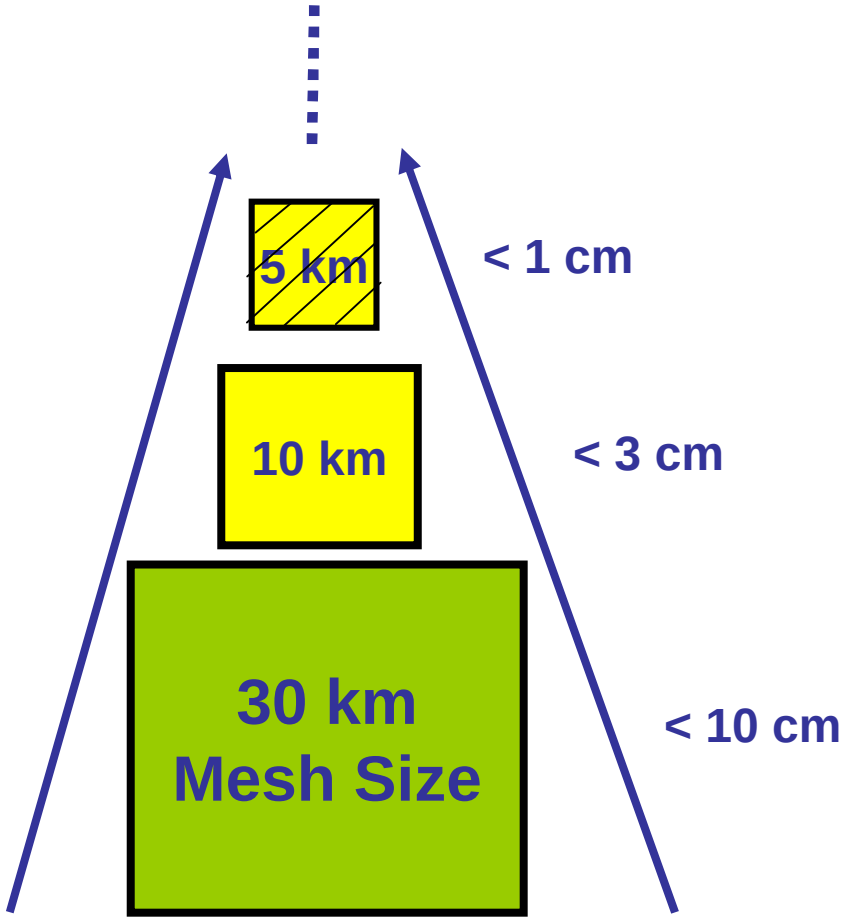
- **EUVN2000 points**

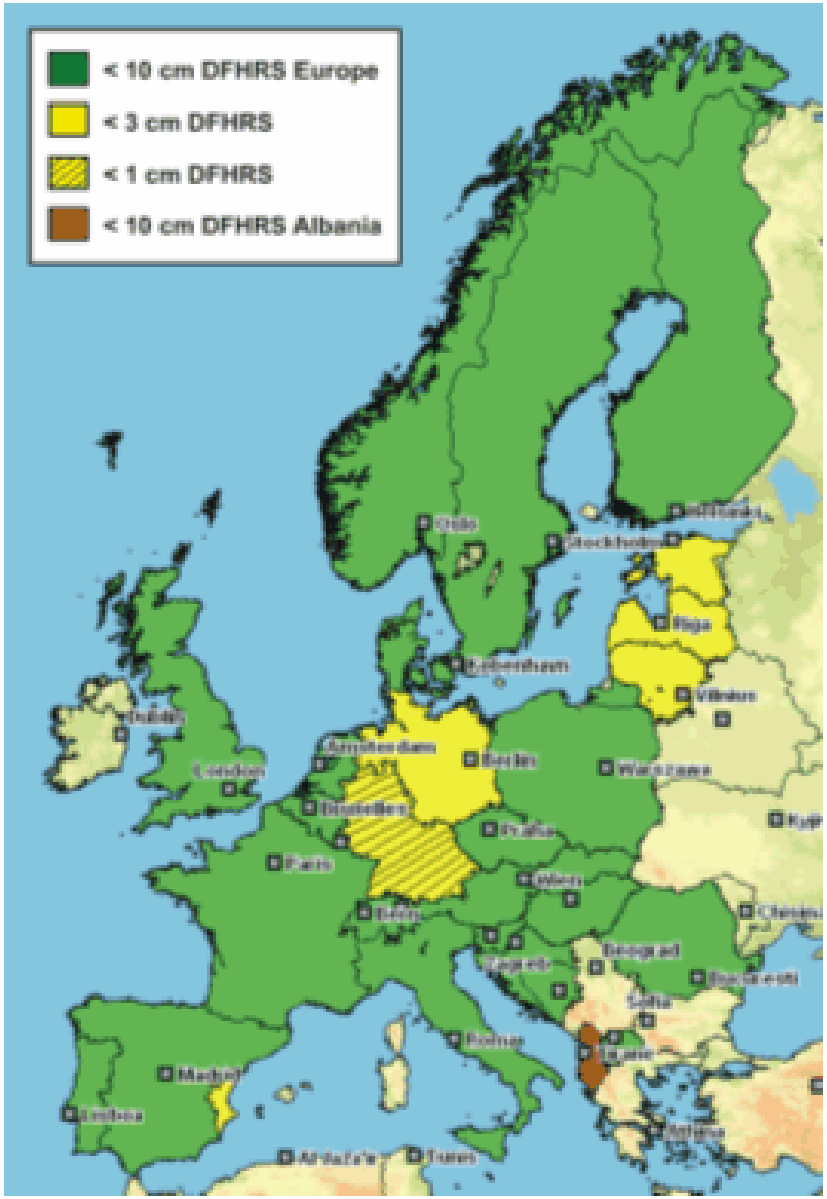
and

- **National EUVN. Some densification points Germany, Estiona, Latvia, Lithuania and Switzerland**



Overview on European DFHRS_DB



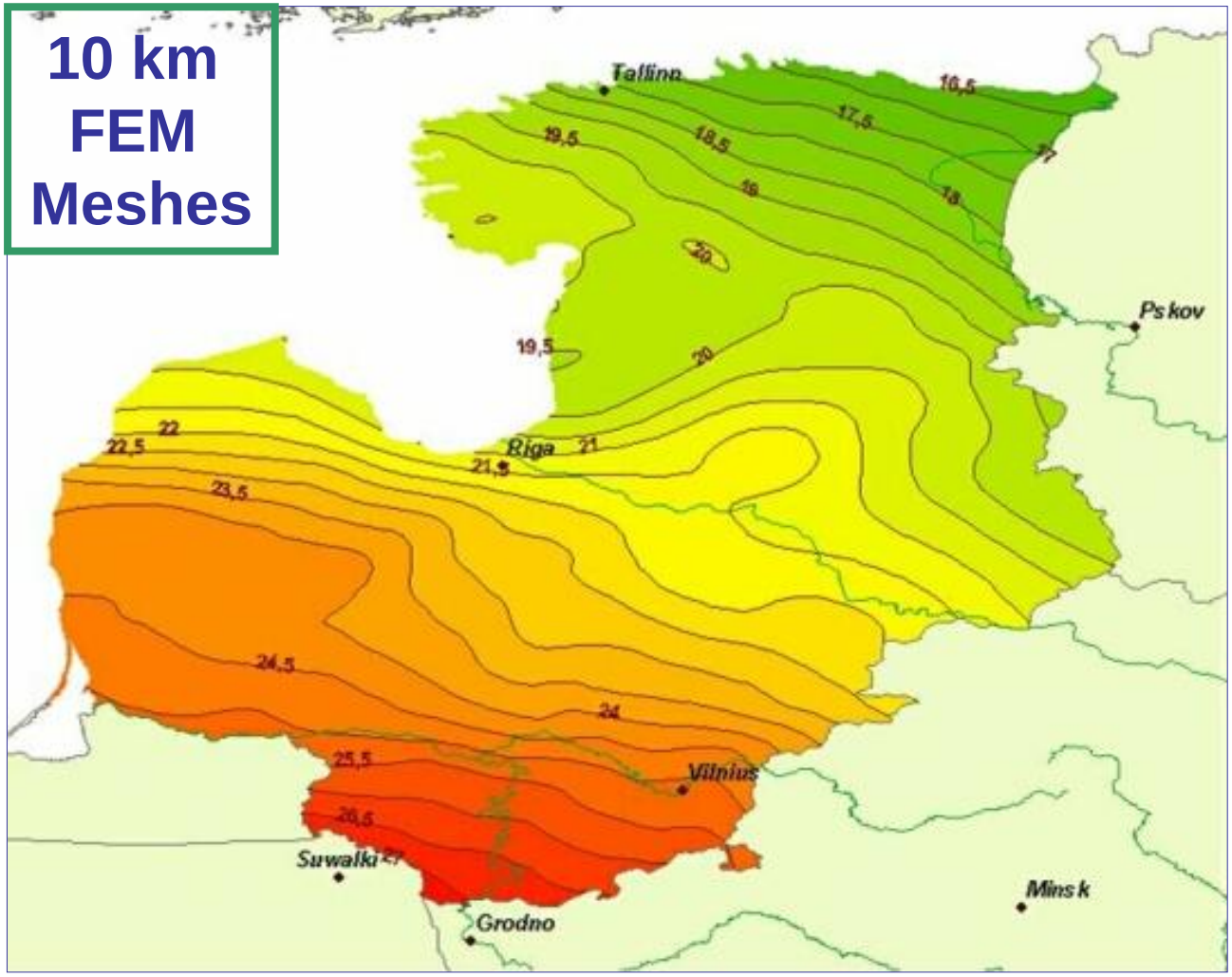


European HRS… including

< 3cm DFHRS_DB Germany

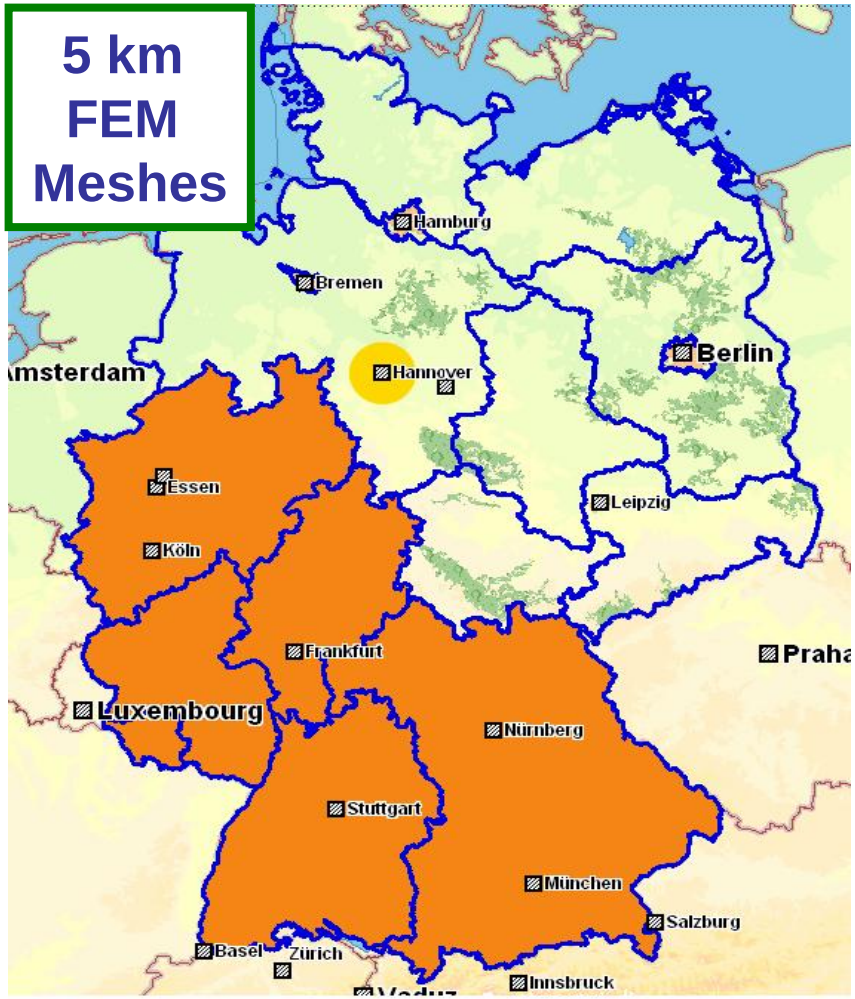
10 km FEM Meshes

European HRS …. including
<(1-3)cm_DFHRS Baltics (Latvia, Estonia, Lithuania)



… including
< 1cm DFHRS_DB Germany

5 km FEM Meshes



< 1 cm DFHBF NN-Höhen
< 5 mm DFHBF Normalhöhen

www.sapos.de

Official State Standard … over years

Abb.: Grobstruktur der DFHBF RP

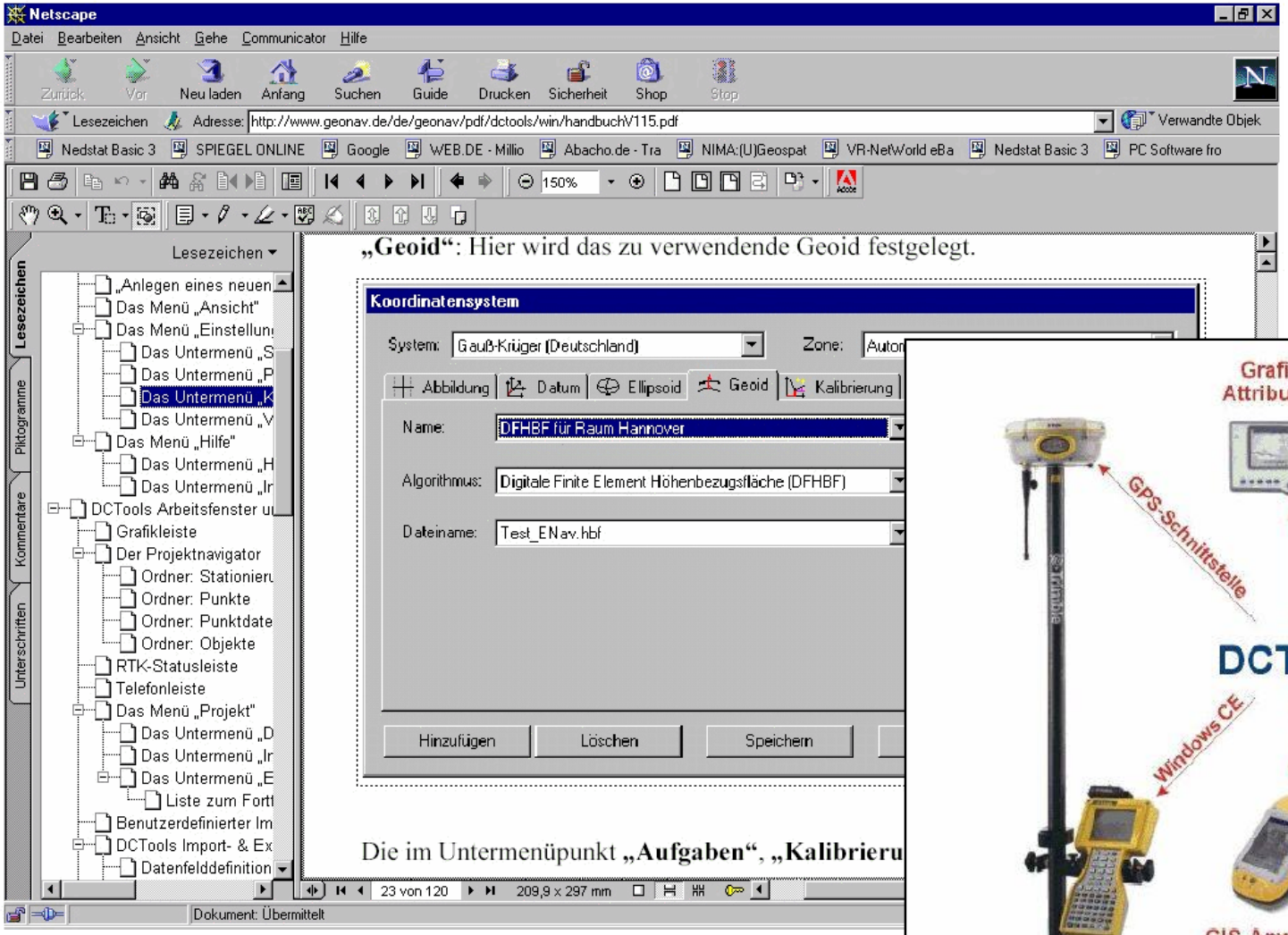
Herausgeber:

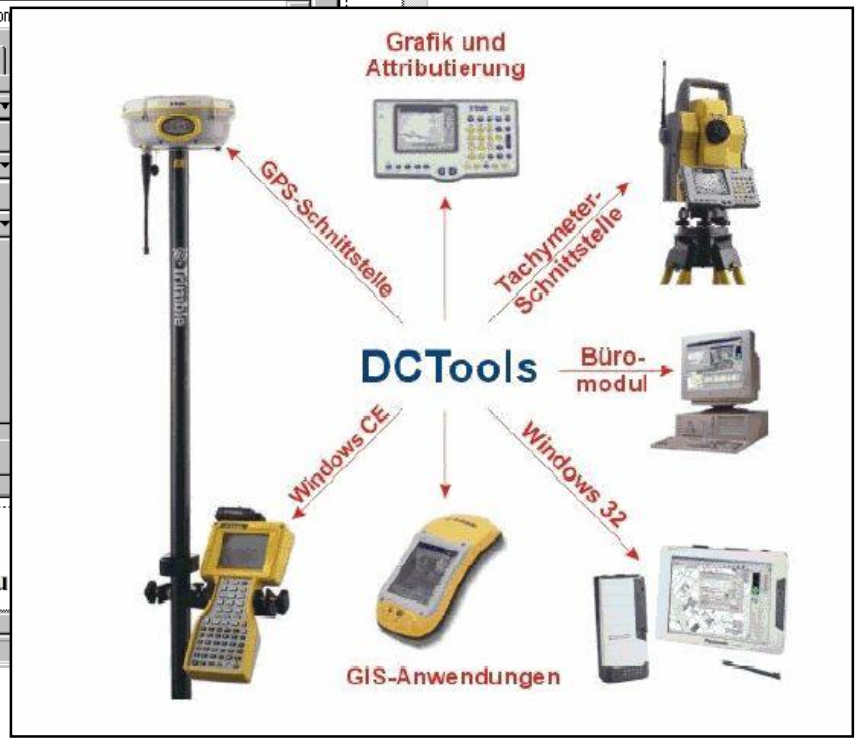
LVermGeo
Landesamt für Vermessung und Geobasisinformation Rheinland-Pfalz

Ferdinand-Sauerbruch-Straße 15
56073 Koblenz
http://www.lvermgeo.rlp.de

DFHRS in Practice

DCTools





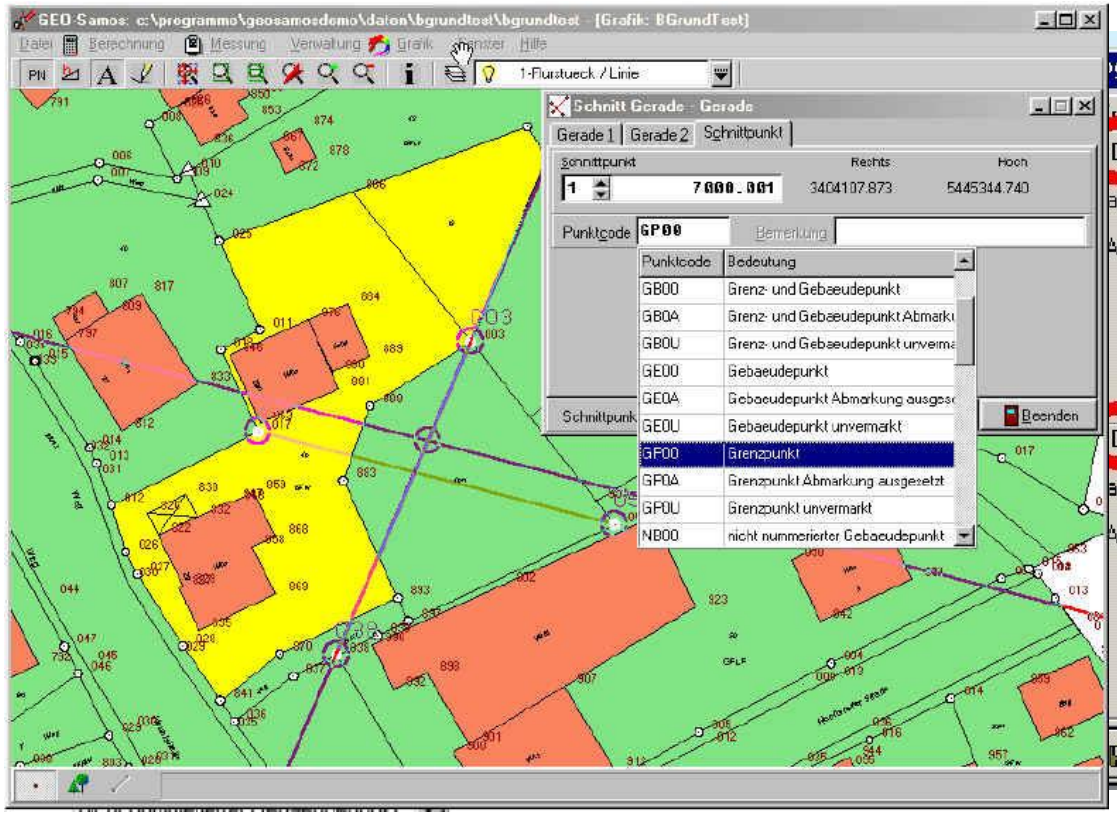
GEONAV
Gesellschaft für geodätische und navigatorische Anwendungen mbH

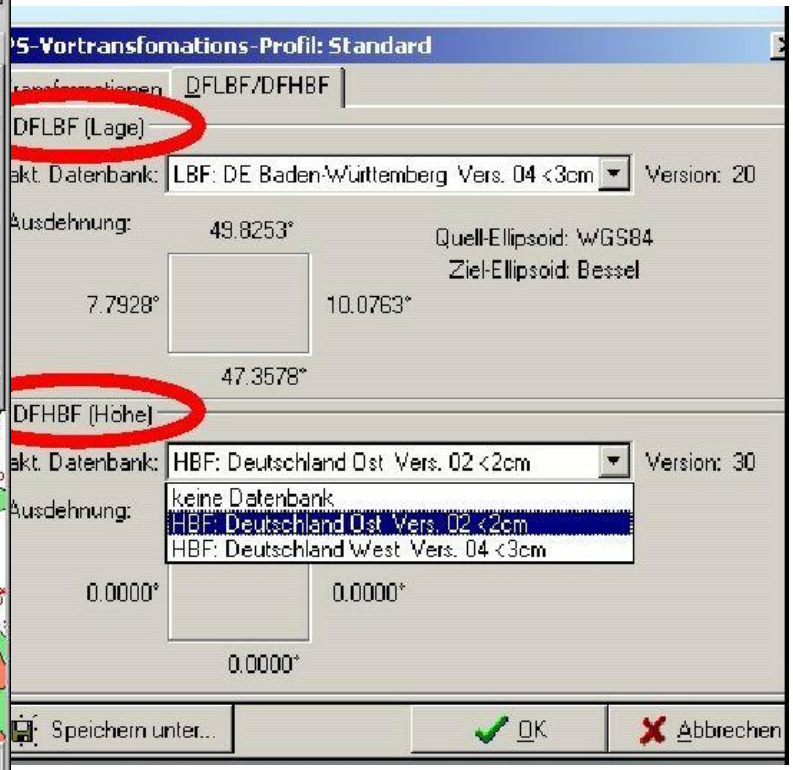
DFHRS in Practice

GEO-Samos

Software for Aktiv Mobile Objectoriented Surveying

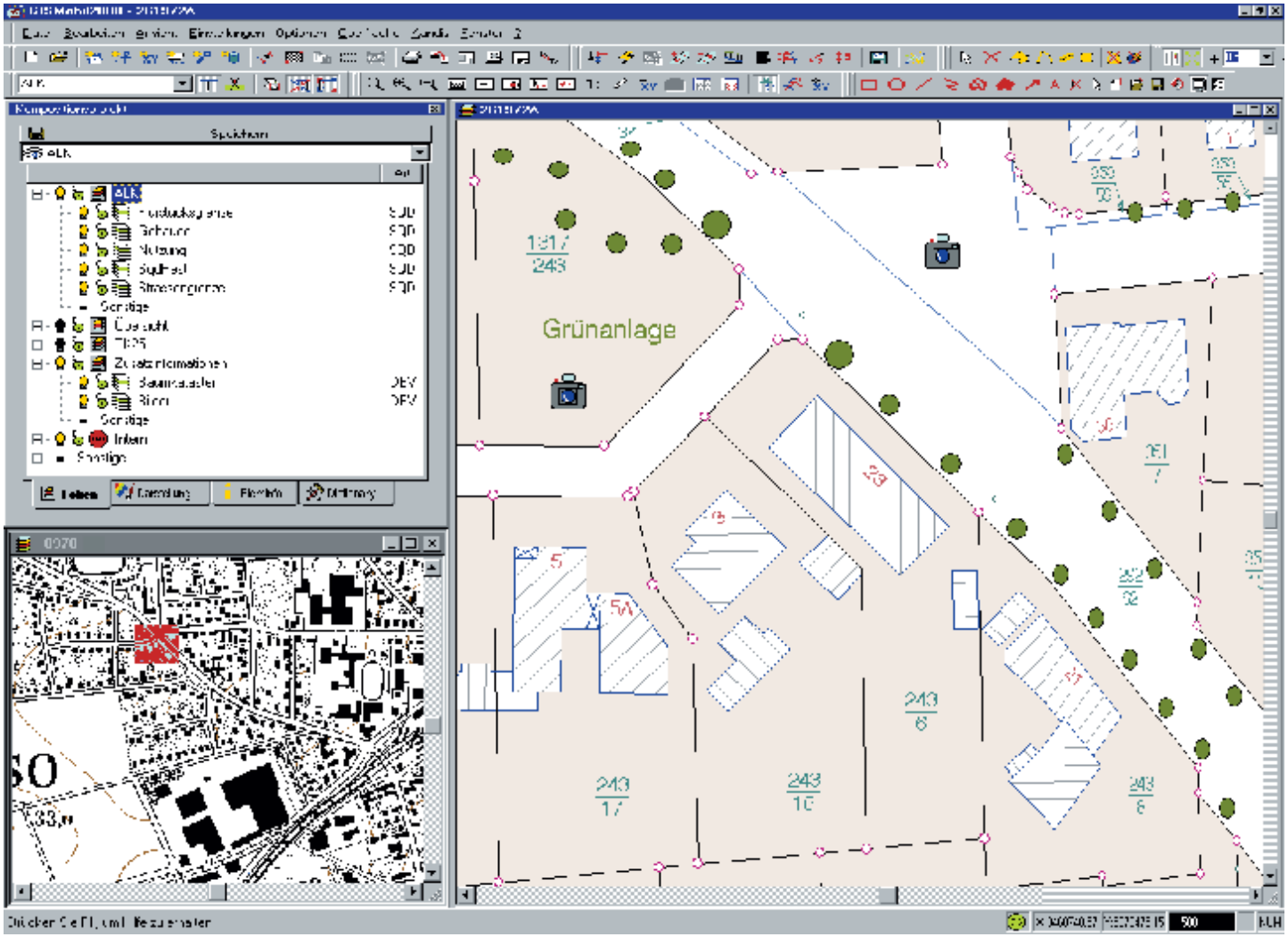
Ingenieurbüro Breining GmbH





DFHRS in Practice

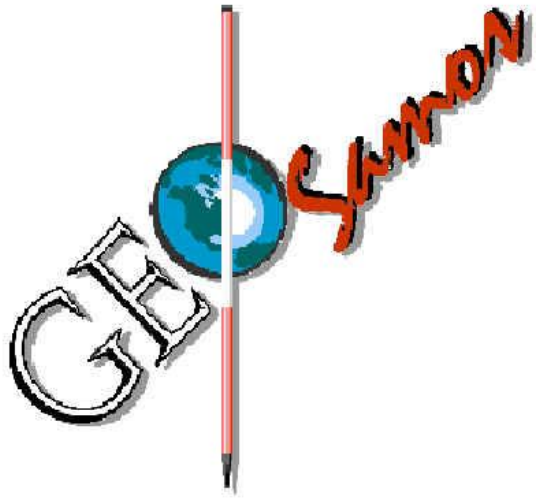
GISMobil



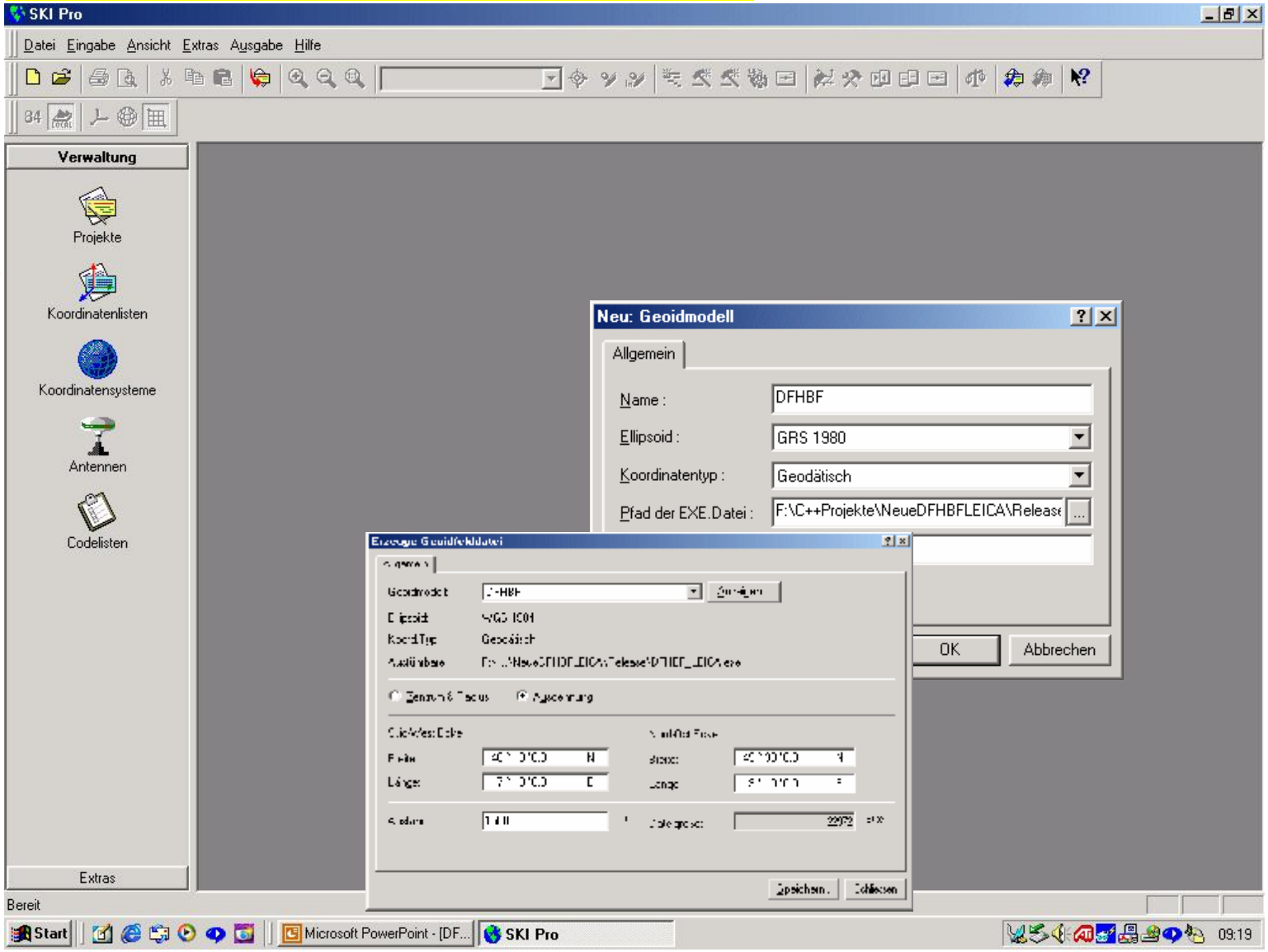
DFHRS in Practice

TOPCON

GART-2000®



DFHRS in Practice



DFHRS in Practice

GPS System 500: Mit V4.0 jetzt noch mehr Möglichkeiten!

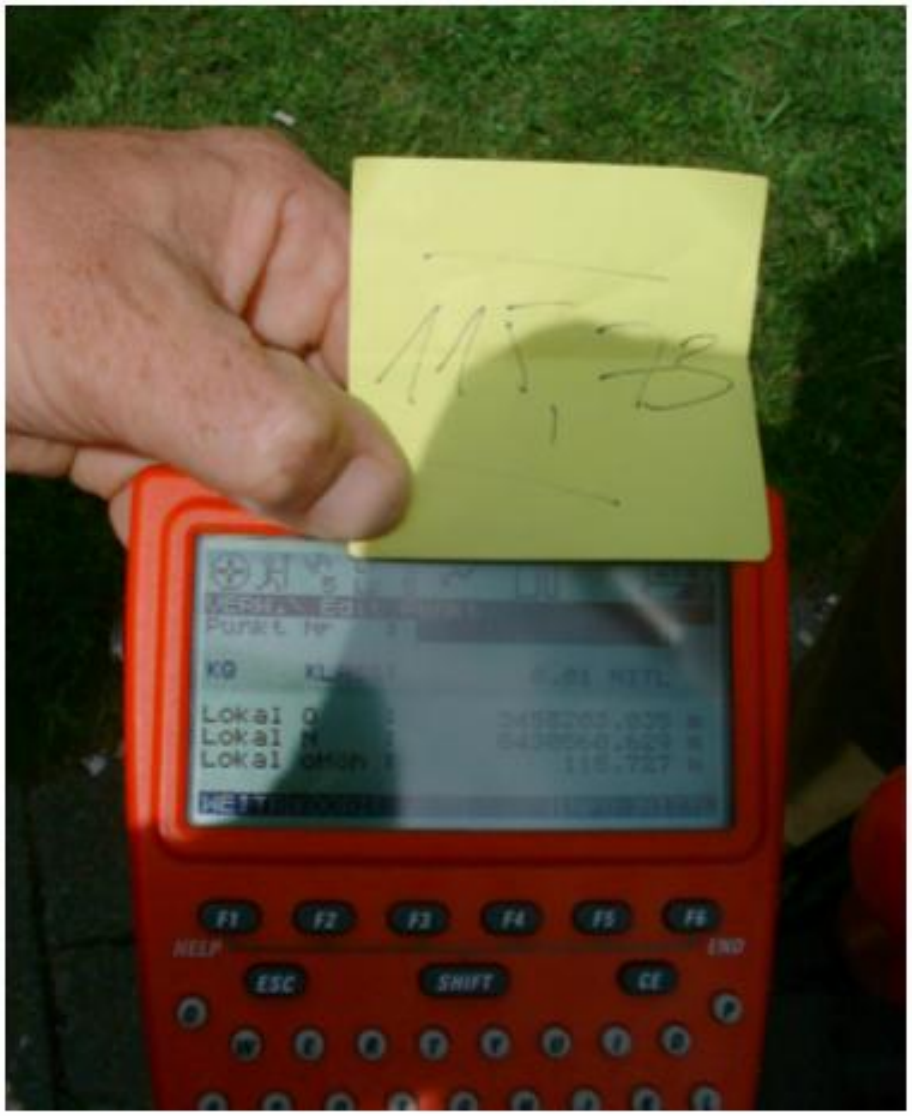
Referenzdienste und Vernetzung:
- Nutzung von SAPOS- und ASCOS-Daten
- Verschiedene Vernetzungslösungen
- Vordefinierte „ADVNULLANTENNA"

Kataster- und Ingenieuranwendung:
- Mittelungslimit absolut oder gewichtet
- Restklaffenverteilung im Feld
- Höhenmodell „DFHBF"
- Identifikation des nächstliegenen Punktes.

Technik:
- Permanen
- RTK Zuver
- Kompatibe
- Koordinat
- Integrierte

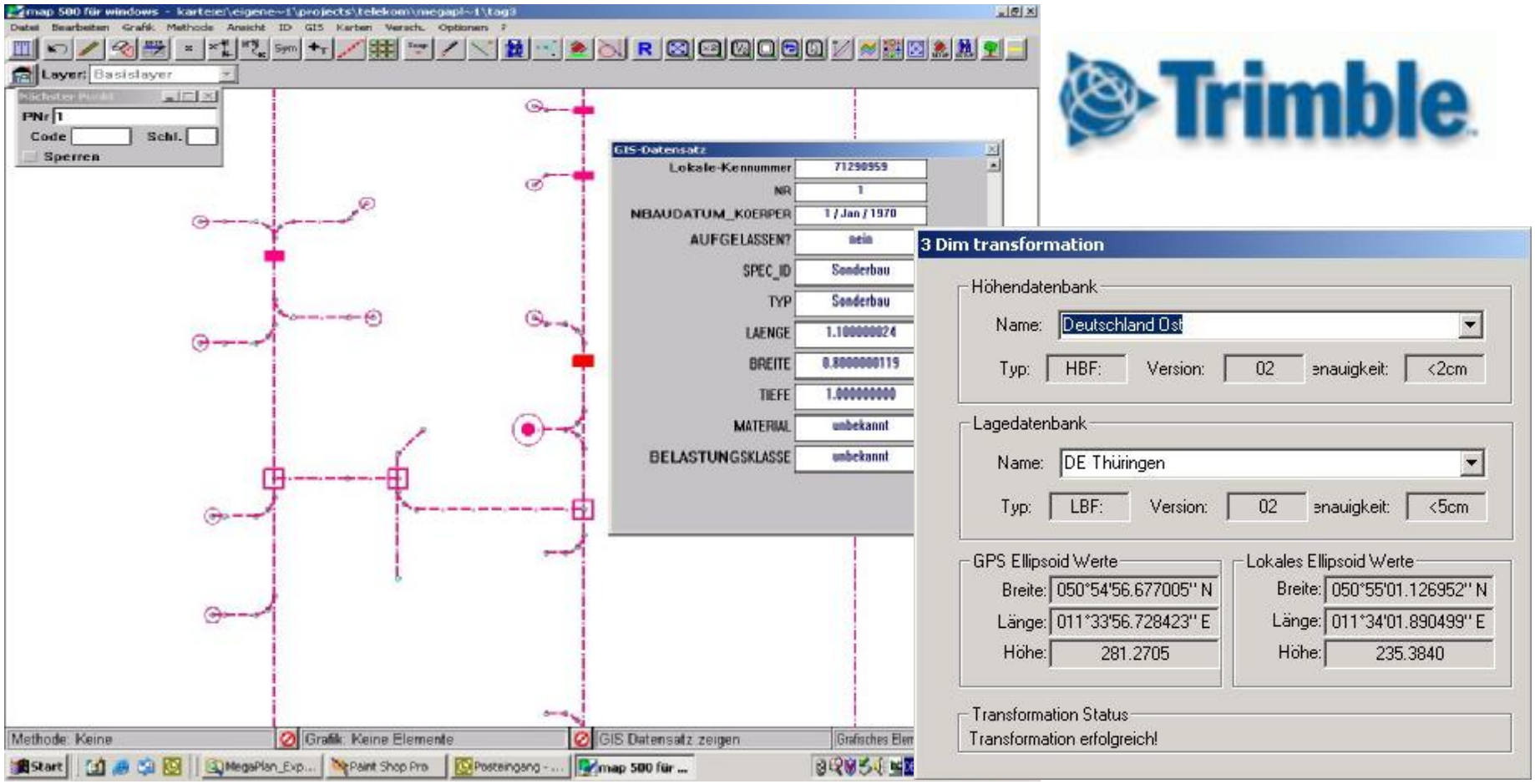
Und viele w

Mit der neu
für jede An

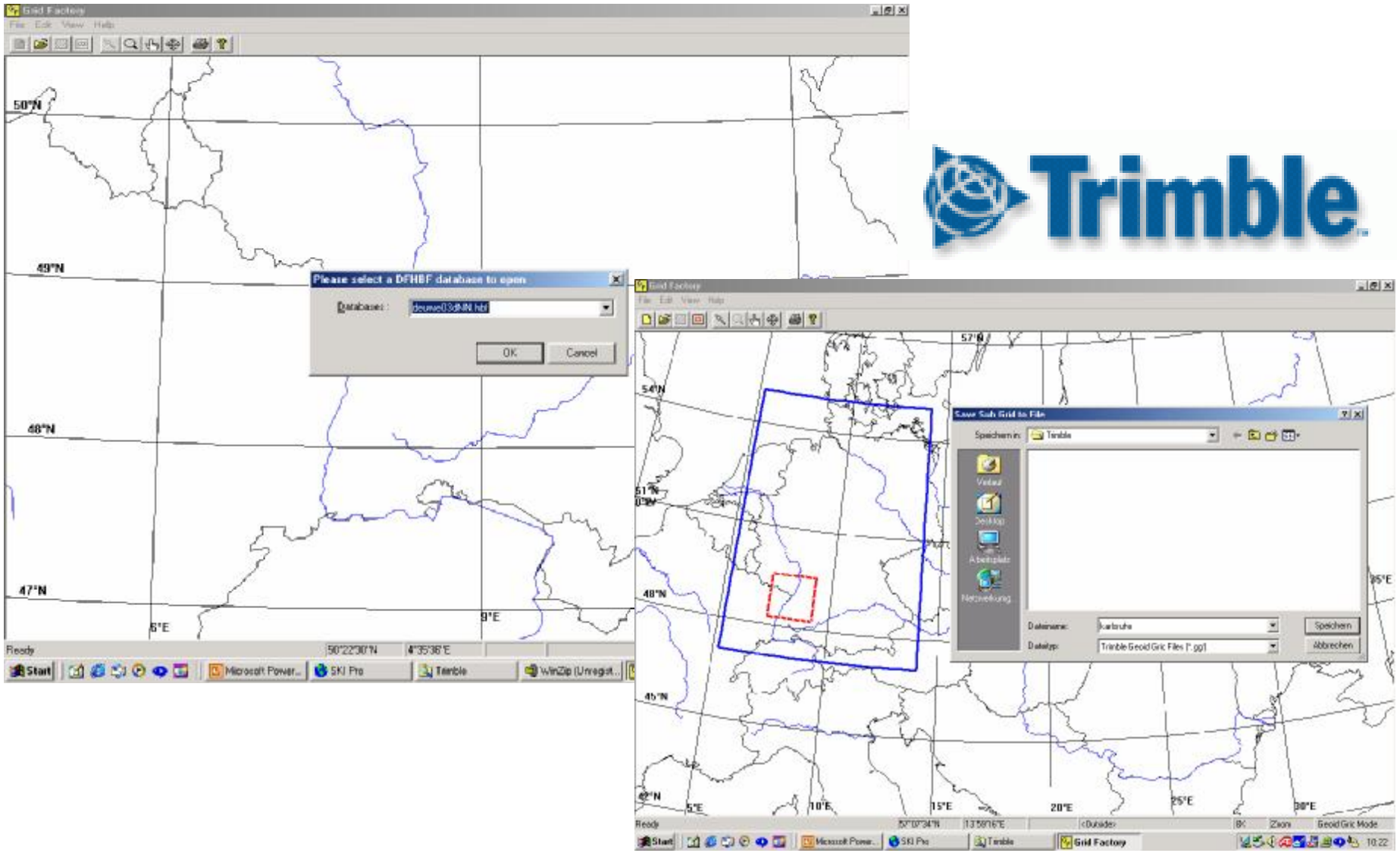


DFHRS in Practice

MAP500



DFHRS in Practice



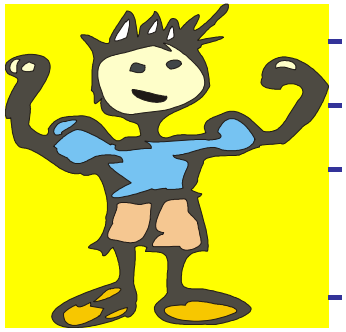
Summary

•GNSS-Age
•GNSS-Positioning

• Spectrum of Fundamental Transformation Problems

DFHBF Concept (Height)

- **Fundamental Solution Concept for HRS**



- Strict mathematical base for a continuous FEM-based HRS
- New concept for an overdetermined parametric HRS
- Mesh and patch-design - Any accuracy and any area size
- Open for all geometrical and physical observations!
- DFHRS = Leading Geoidfitting Concept
- Ready for the < 1 cm EVRS using all existing data! and EPN densifications!
- High Practical Relevance for GNSS services and GIS
- Industrial Standard GNSS-Equipment and GIS (+ RTCM 3.0)
- High Capacities for International Co-operations